

Process Capability

Notebook Tab 6 - Pages 254 to 268

CQE Handbook pages 527 to 533

G. **Process and Performance Capability**

1. **Process capability studies**

Define, describe, calculate, and use process capability studies, including identifying characteristics, specifications, and tolerances, developing sampling plans for such studies, establishing statistical control, etc. (Analyze)

2. **Process performance vs. specifications**

Distinguish between natural process limits and specification limits, and calculate percent defective. (Analyze)

3. **Process capability indices**

Define, select, and calculate C_p , C_{pk} , C_{pm} , and C_r , and evaluate process capability. (Evaluate)

4. **Process performance indices**

Define, select, and calculate P_p and P_{pk} and evaluate process performance. (Evaluate)

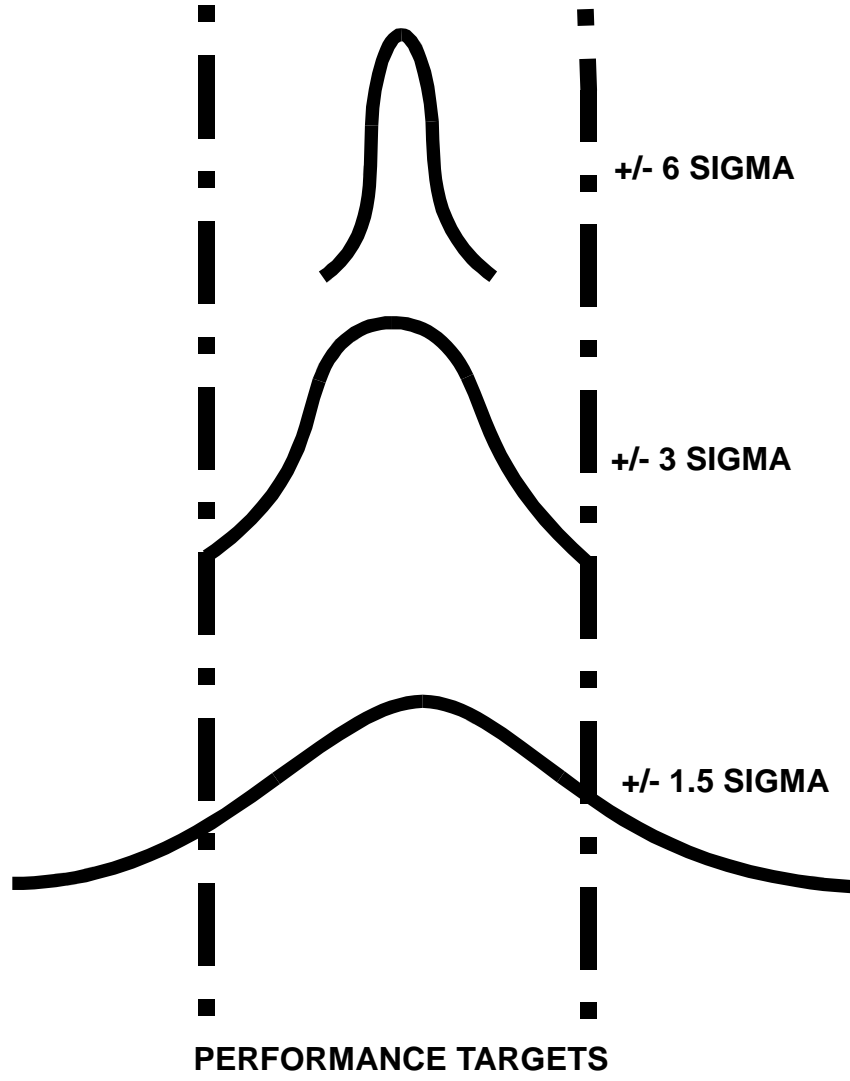
Comparison of Common Process Capability Measures

- *Percent defective* - The percent of product that is nonconforming or defective.
- *Process yield* - The percent of product that meets its requirements.
- *PPM / DPMO* - When the number of defective products produced is small, it is often shown as a ratio of the number of defectives in 1 million parts or the number of defects in 1 million opportunities. This is referred to as “PPM” for “parts per million defective” or “DPMO” for “defects per million opportunities.”

For example, a process that is producing 99.73% good product is producing .27% defective product, or 2700 parts per million (PPM) defective.

- *Process Capability Indices (Cp, Cpk)* – Measure the relationship of the process “bell curve” to the specification limits; use “short-term” / within-group variation.
- *Process Performance Indices (Pp, Ppk)* – Measure the relationship of the process “bell curve” to the specification limits; use “long-term” / between-group variation.
- *Sigma Levels* - A sigma level is nothing more than a z-score – it measures the number of standard deviations of the process that can fit between the process average and the *nearest* performance limit or target.

For example, if a particular process’ average is 3 standard deviations from the nearest performance limit, the process is operating at a “3 sigma” level. A “6 sigma” level would mean that the process is centered six standard deviations away from the nearest limit.



- A z-score is nothing more than a distance measure - in all cases it represents the number of standard deviations between the mean of the data set and some target value or point of interest.
- If the point of interest is the nearest specification limit, the z score can be called a Sigma level!

$$Z = \frac{X - \mu}{\sigma}$$

z Score / Sigma level Example:

- The process is stable!!
- The process is normally distributed!!
- Process average = 12.24
- Process standard deviation = 1.13759
- Upper spec limit = 14
- Lower spec limit = 10

$$Z = \frac{X - \mu}{\sigma}$$

What is the z score / sigma level for the upper spec limit?

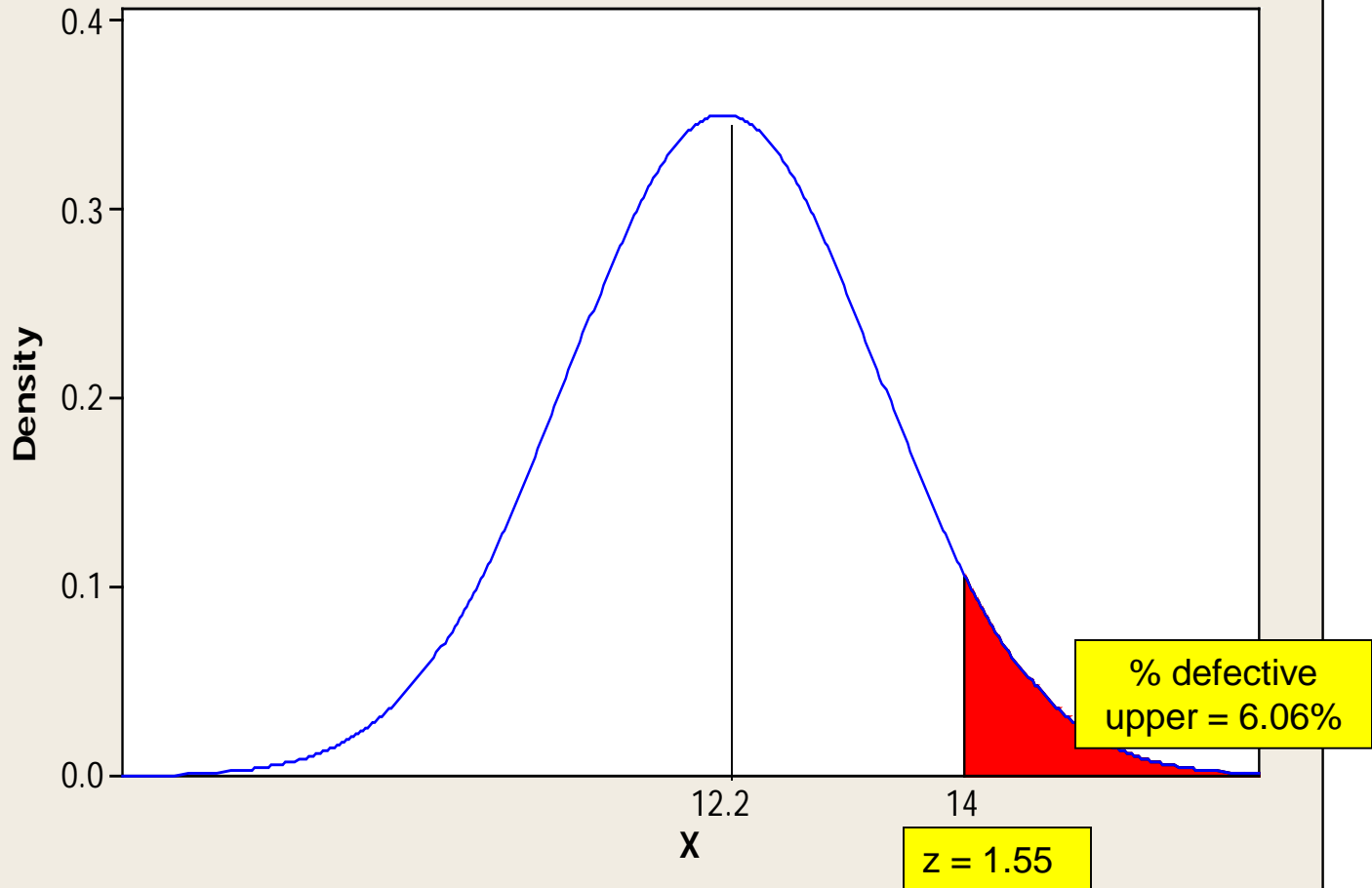
z Score / Sigma level Example:

- The process is stable!!
- The process is normally distributed!!
- Process average = 12.24
- Process standard deviation = 1.13759
- Upper spec limit = 14
- Lower spec limit = 10

$$Z = \frac{x - \mu}{\sigma} = \frac{14 - 12.24}{1.13759} = 1.55$$

Distribution Plot

Normal, Mean=12.24, StDev=1.13759



z Score / Sigma level Example:

- The process is stable!!
- The process is normally distributed!!
- Process average = 12.24
- Process standard deviation = 1.13759
- Upper spec limit = 14
- Lower spec limit = 10

$$Z = \frac{X - \mu}{\sigma}$$

What is the z score / sigma level for the lower spec limit?

z Score / Sigma level Example:

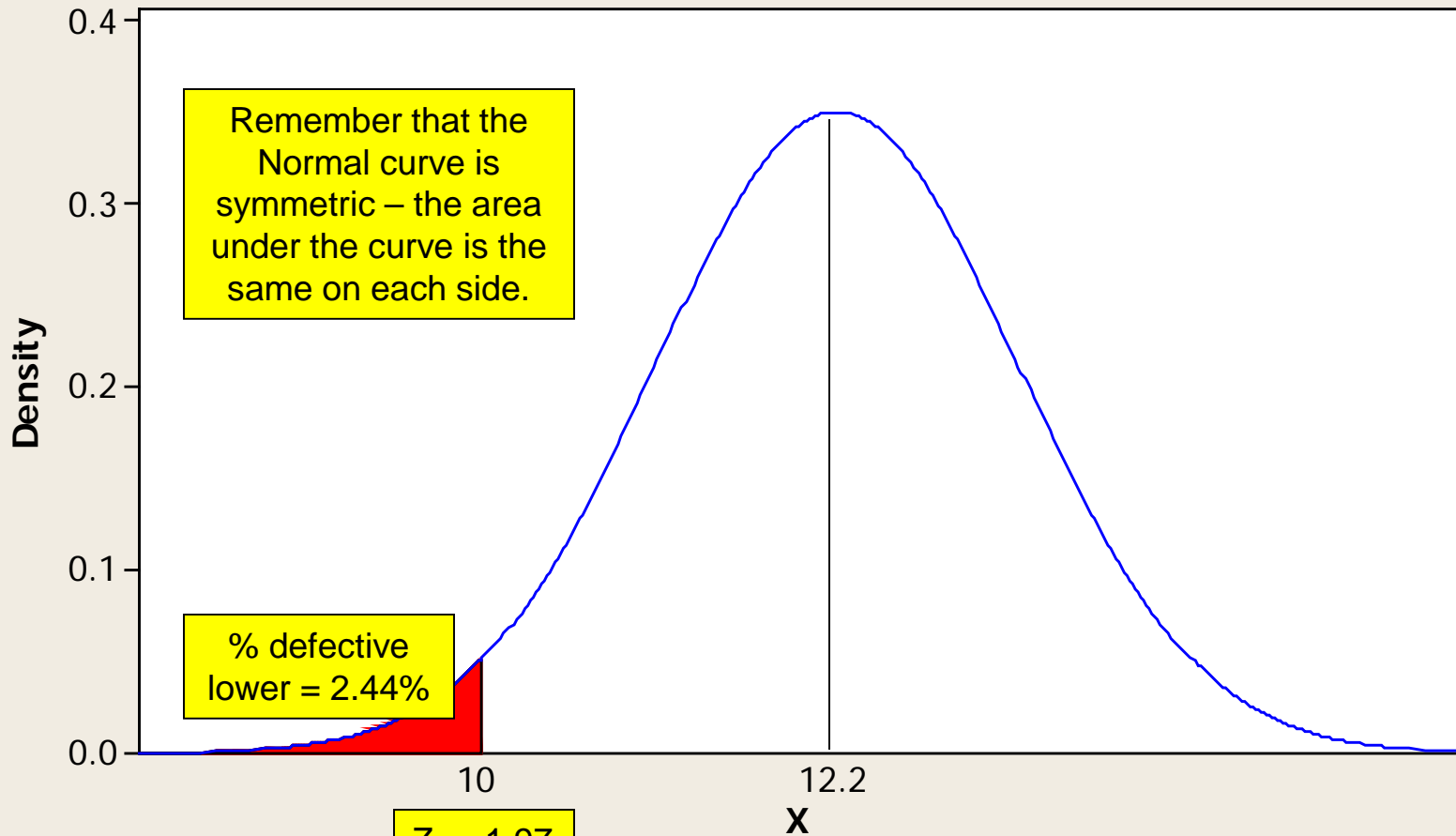
- The process is stable!!
- The process is normally distributed!!
- Process average = 12.24
- Process standard deviation = 1.13759
- Upper spec limit = 14
- Lower spec limit = 10

**What is the z score /
sigma level for the
lower spec limit?**

$$z = \frac{x - \mu}{\sigma} = \frac{10 - 12.24}{1.13759} = -1.97$$

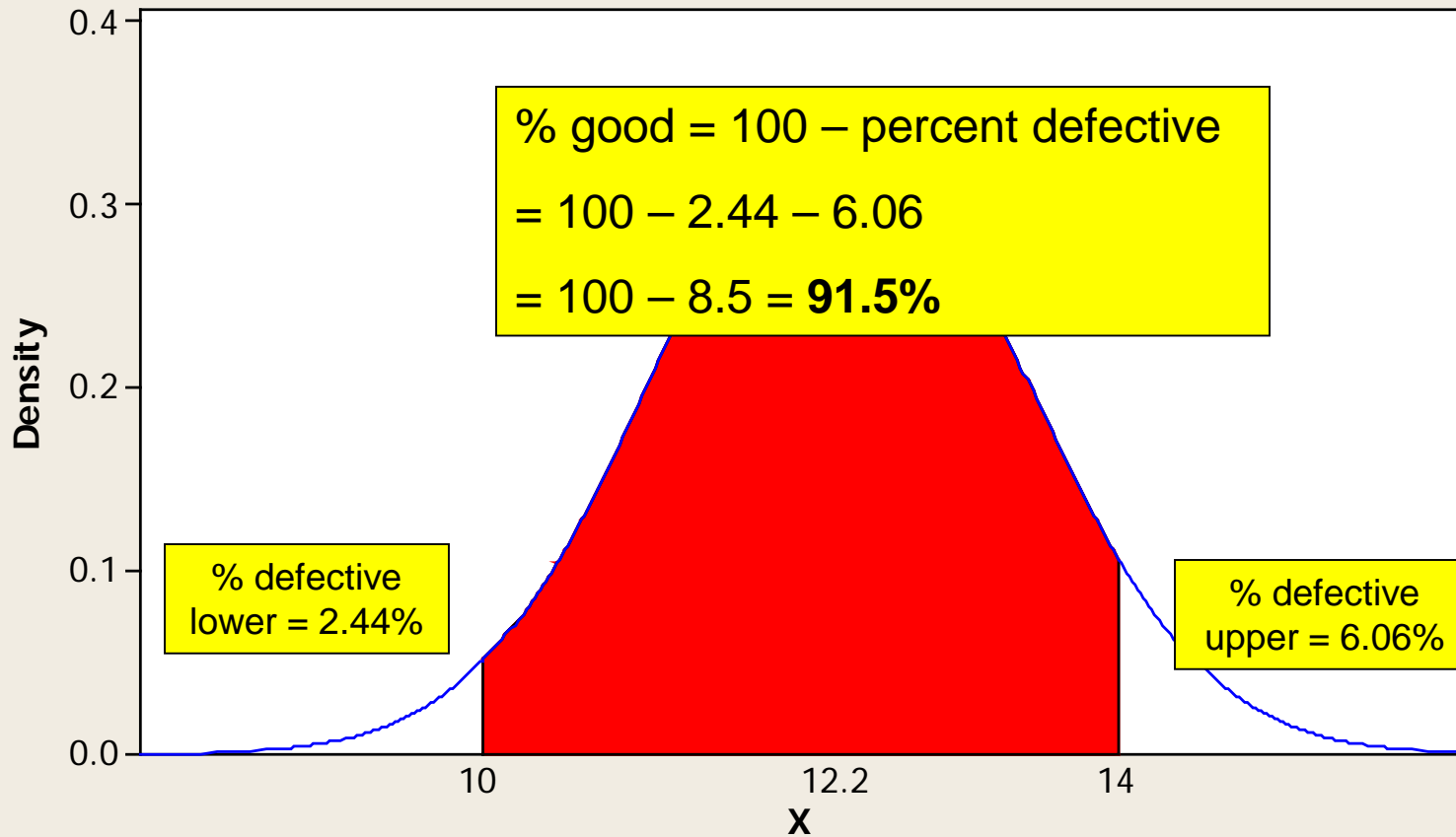
Distribution Plot

Normal, Mean=12.24, StDev=1.13759



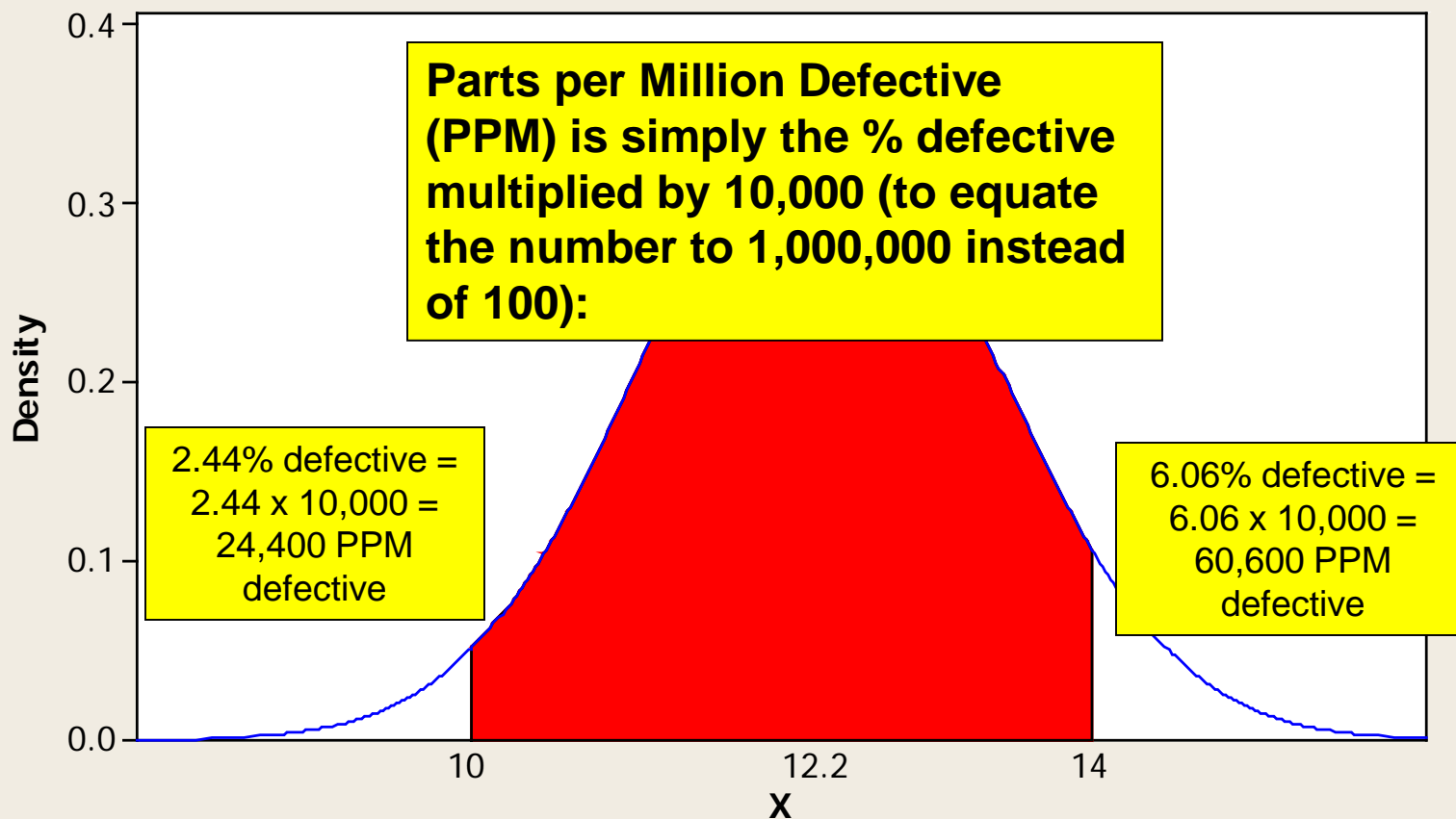
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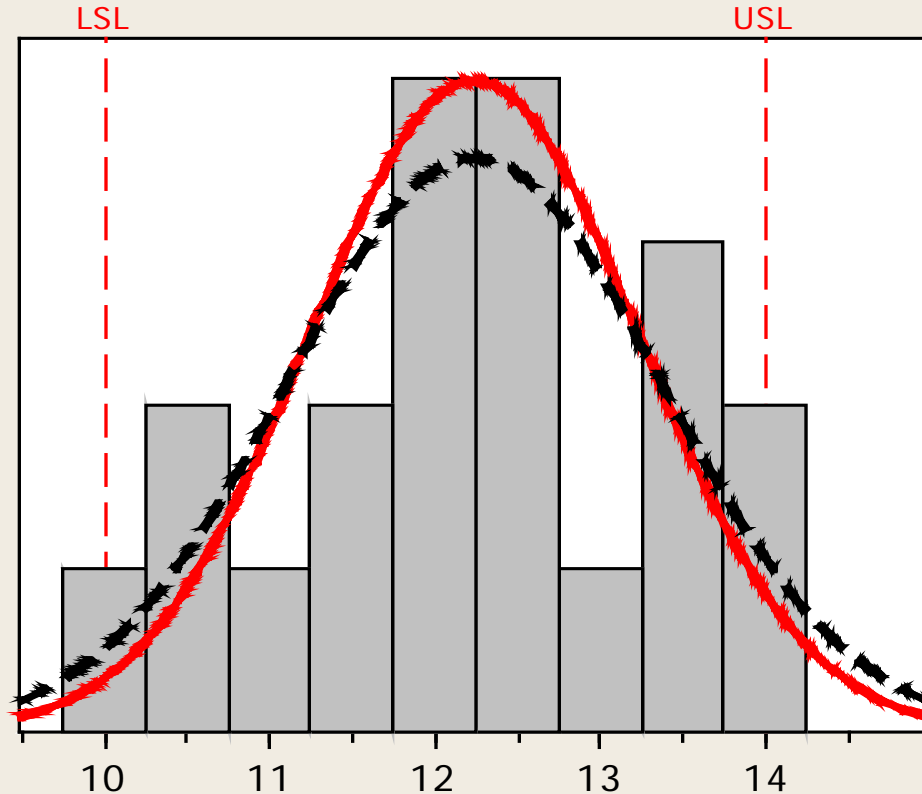
Distribution Plot

Normal, Mean=12.24, StDev=1.13759



Process Capability of Data

Process Data	
LSL	10
Target	*
USL	14
Sample Mean	12.24
Sample N	20
StDev (Within)	1.00141
StDev (Overall)	1.13759



—	Within
- - -	Overall

Potential (Within) Capability	
Z.Bench	1.63
Z.LSL	2.24
Z.USL	1.76
Cpk	0.59

Overall Capability	
Z.Bench	1.37
Z.LSL	1.97
Z.USL	1.55
Ppk	0.52
Cpm	*

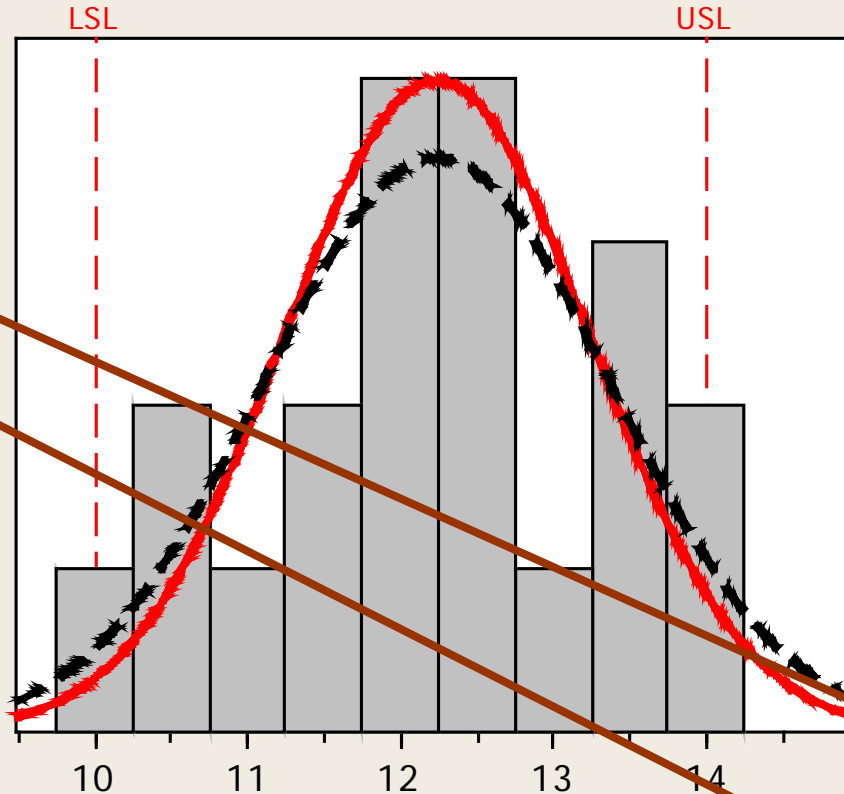
Observed Performance	
PPM < LSL	0.00
PPM > USL	50000.00
PPM Total	50000.00

Exp. Within Performance	
PPM < LSL	12648.50
PPM > USL	39415.05
PPM Total	52063.54

Exp. Overall Performance	
PPM < LSL	24471.99
PPM > USL	60915.50
PPM Total	85387.49

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Cpm	*

“Within” variation is based on the Range / MR / Sigma Chart average

“Overall” variation is based on the standard deviation across all results

Observed Performance	
PPM < LSL	0.00
PPM > USL	50000.00
PPM Total	50000.00

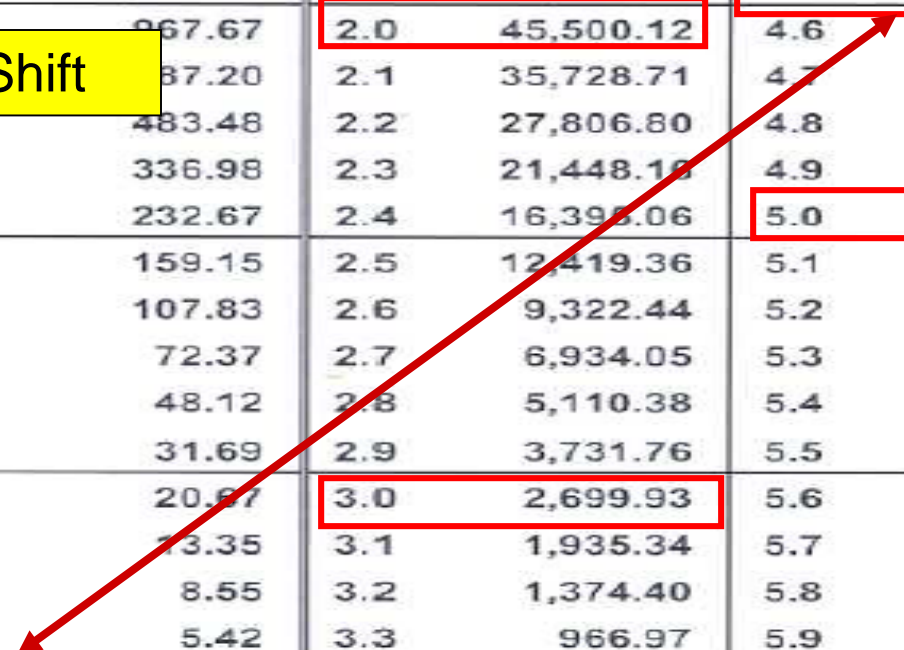
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<u>SIGMA LEVEL</u> <u>(z SCORE)</u>		<u>PERCENT OF PROCESS WHICH MEETS REQUIREMENTS</u> (With the process centered)	<u>MISTAKES PER MILLION ATTEMPTS (PPM)</u> (With the process centered)	<u>Cp / Cpk</u> (With the process centered)
+/- 1 sigma	=	68.26894805%	317,310.520 (~317,000)	0.33
+/- 2 sigma	=	95.44998759%	45,500.124 (~ 45,500)	0.67
+/- 3 sigma	=	99.73000656%	2699.934 (~ 2700)	1.00
+/- 4 sigma	=	99.99366279%	63.372 (~ 63)	1.33
+/- 5 sigma	=	99.99994257%	.574 (~ 0.6)	1.67
+/- 6 sigma	=	99.999999801976%	.00198024 (~ 2 PPB!)	2.00

With a 1.5 σ Process Shift				With No Process Shift			
Z	ppm	Z	ppm	Z	ppm	Z	ppm
1.0	697,672.15	3.6	17,864.53	1.0	317,310.52	3.6	318.29
1.1	660,082.92	3.7	13,903.50	1.1	271,332.20	3.7	215.66
1.2	621,378.38	3.8	10,724.14	1.2	230,139.46	3.8	144.74
1.3	581,814.88	3.9	8,197.56	1.3	193,601.10	3.9	96.23
1.4	541,693.78	4.0	6,209.70	1.4	161,513.42	4.0	63.37
1.5	501,349.97	4.1	4,661.23	1.5	133,614.46	4.1	41.34
1.6	461,139.78	4.2	3,467.03	1.6	109,598.58	4.2	26.71
1.7	421,427.51	4.3	2,555.19	1.7	89,130.86	4.3	17.09
1.8	382,572.13	4.4	1,865.88	1.8	71,860.53	4.4	10.83
1.9	344,915.28	4.5	1,349.97	1.9	57,432.99	4.5	6.80
2.0	308,770.21	4.6	967.67	2.0	45,500.12	4.6	4.23
2.1	274,837.20	4.7	687.20	2.1	35,728.71	4.7	2.60
2.2	242,071.41	4.8	483.48	2.2	27,806.80	4.8	1.59
2.3	211,927.71	4.9	336.98	2.3	21,448.10	4.9	0.960
2.4	184,108.21	5.0	232.67	2.4	16,395.06	5.0	0.574
2.5	158,686.95	5.1	159.15	2.5	12,419.36	5.1	0.340
2.6	135,686.77	5.2	107.83	2.6	9,322.44	5.2	0.200
2.7	115,083.09	5.3	72.37	2.7	6,934.05	5.3	0.116
2.8	96,809.10	5.4	48.12	2.8	5,110.38	5.4	0.067
2.9	80,762.13	5.5	31.69	2.9	3,731.76	5.5	0.038
3.0	66,810.63	5.6	20.87	3.0	2,699.93	5.6	0.021
3.1	54,801.40	5.7	13.35	3.1	1,935.34	5.7	0.012
3.2	44,566.73	5.8	8.55	3.2	1,374.40	5.8	0.007
3.3	35,931.06	5.9	5.42	3.3	966.97	5.9	0.004
3.4	28,716.97	6.0	3.40	3.4	673.96	6.0	0.002
3.5	22,750.35	6.1	2.11	3.5	465.35	6.1	0.001

The 1.5 Sigma Motorola Shift



Process Capability Indices: C_p , C_{pk} (Or C_{p_u} And C_{p_l})

- The C_p and C_{pk} family of process capability indices *predict* the *potential capability* of the process to meet its requirements - they reflect how the process *could* perform if the shifts and drifts of the process were to be eliminated.

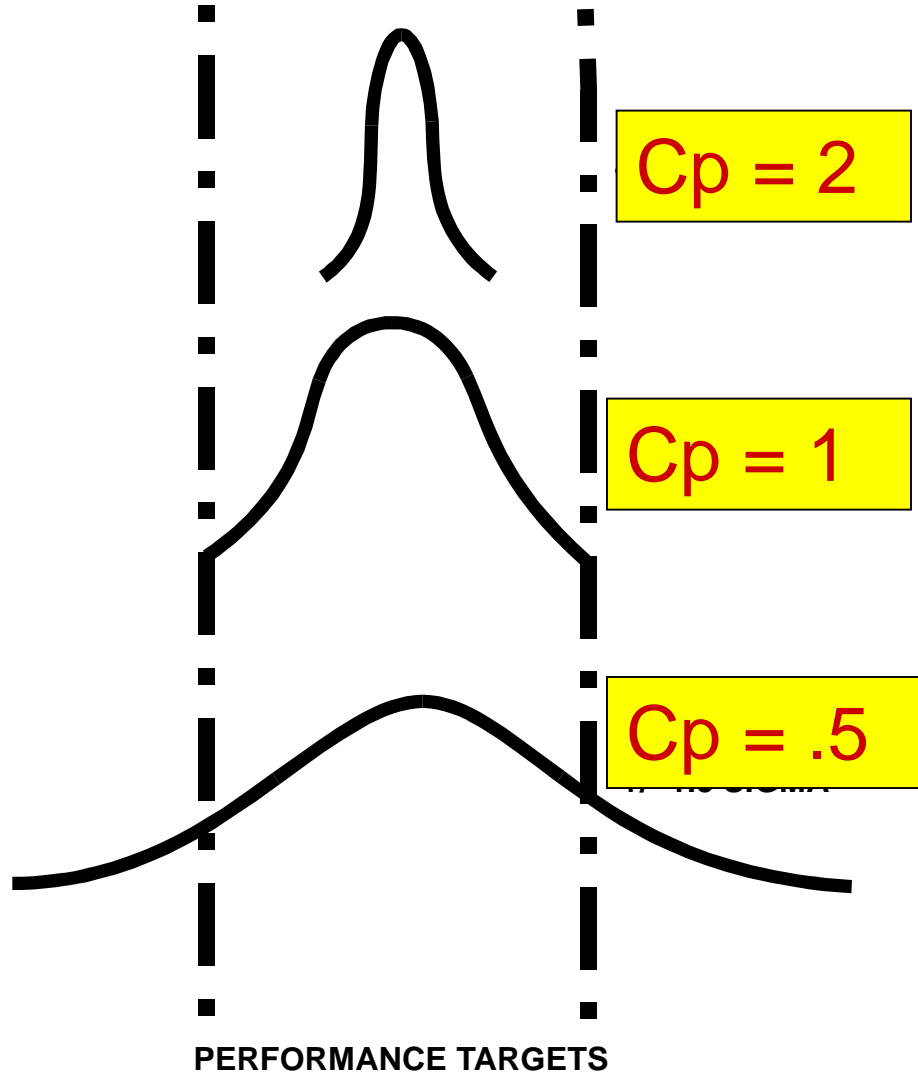
There is disagreement over the use of the term “short” term vs. “long” term:

- Some people also refer to the C_p and C_{pk} as the “*potential*” or “*long-term*” process capability indices because they reflect how the process *could* perform if it were completely stable.
- Others call these the “*short term*” indices because they are based on the average amount of variation within each sample group, which is a smaller period of time than the entire data set.
- Originally called t index in Shewhart work

- The Cp compares the *total predicted process variation* (defined as +/- 3 standard deviations) to the *allowable process variation* (*specification range*):

$$C_p = \frac{\text{Upper Performance Limit} - \text{Lower Performance Limit}}{6 \hat{\sigma}}$$

- Cp therefore measures the *number of Normal curves that will fit within the specification limits*. For example, a Cp of 2 means that the specification range is twice as wide as the total predicted process variation, or that two Normal curves will fit within the specification range.
- Cp can not be computed if we only have one specification limit
- Noted as NA or Non-Applicable



- Because these indices are being used to predict potential process performance, the standard deviation used is always estimated from the Range (or Sigma or Moving Range) control chart as follows:

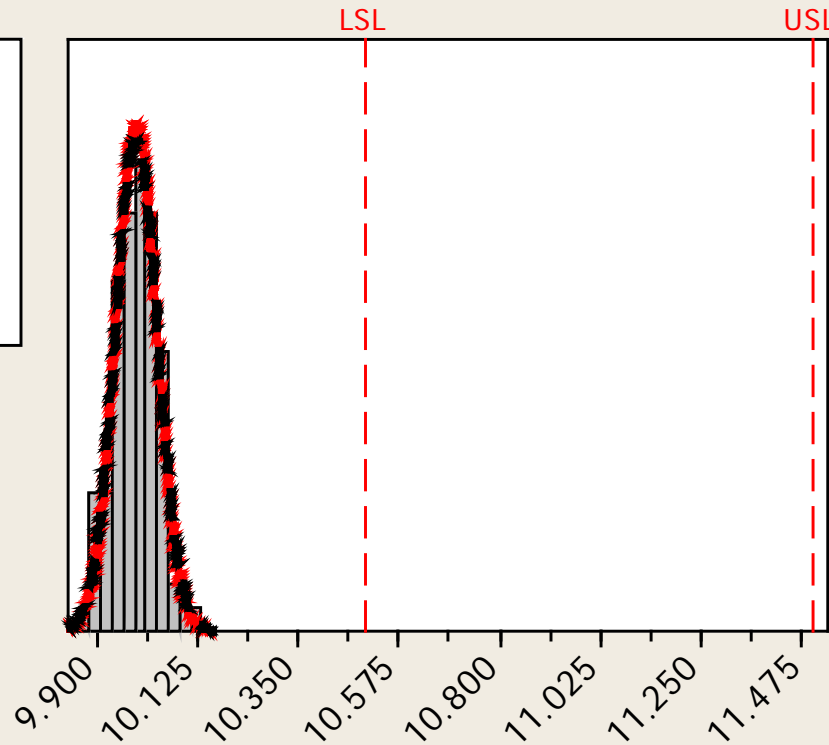
$$\hat{\sigma} = \frac{\bar{R}}{d_2}$$

- \bar{R} is the centerline value for the accompanying Range chart
- d_2 is a calculation factor based on sample size and is found on the following table:

- The C_p is limited in that it does not consider the location of the center of the process distribution.
- A process centered outside the specification limits, and therefore highly defective, could still score a very good C_p if its variation is small enough.
- The C_p is therefore considered to be a preliminary measure only. If the amount of variation is acceptable, then the C_{pk} must be calculated to assess the centering of the process relative to the performance limits.

Process Capability of C1

Process Data	
LSL	10.5
Target	*
USL	11.5
Sample Mean	9.99089
Sample N	100
StDev (Within)	0.0458766
StDev (Overall)	0.0473643



—	Within
—	Overall

Potential (Within) Capability	
Cp	3.63
CPL	-3.70
CPU	10.96
Cpk	-3.70

Overall Capability	
Pp	3.52
PPL	-3.58
PPU	10.62
Ppk	-3.58
Cpm	*

Observed Performance	
PPM < LSL	1000000.00
PPM > USL	0.00
PPM Total	1000000.00

Exp. Within Performance	
PPM < LSL	1000000.00
PPM > USL	0.00
PPM Total	1000000.00

Exp. Overall Performance	
PPM < LSL	1000000.00
PPM > USL	0.00
PPM Total	1000000.00

- The Cpk compares the *actual process center and spread* to the *nominal or target process center and spread*. Cpk is based on the distance from the process mean to the nearest, and therefore riskiest, performance target, so the smallest value is always selected. Comparison of **VOP and VOC**

Cpk = the smaller of:
$$\frac{\text{Upper Performance Limit} - \bar{x}}{3 \hat{\sigma}}$$

or

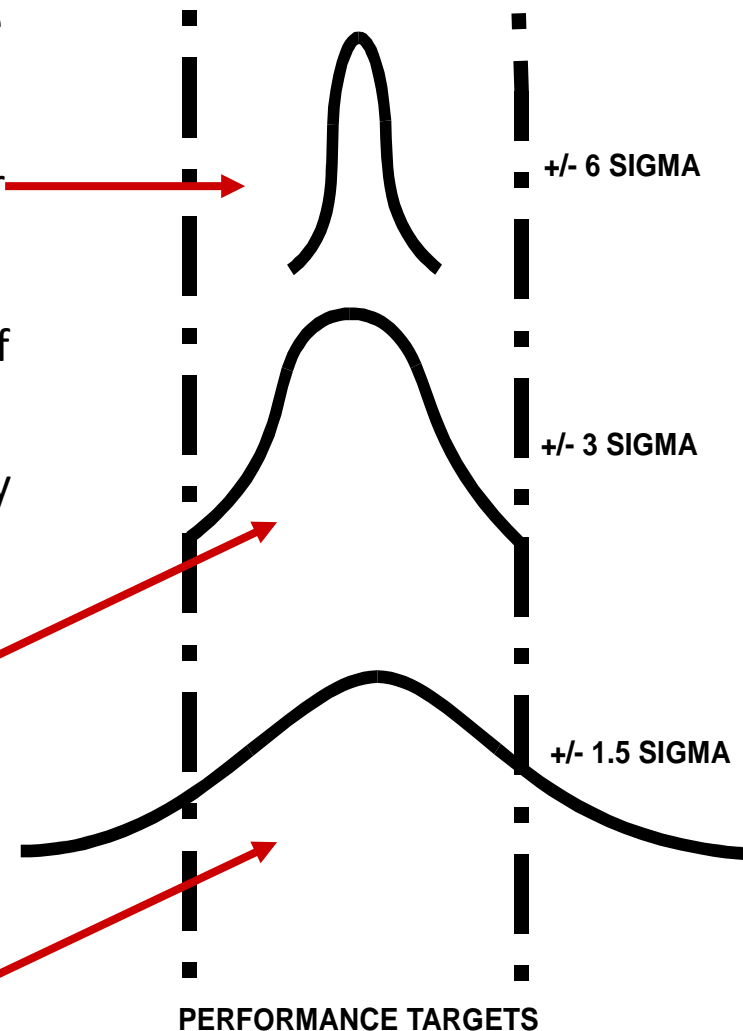
$$\frac{\bar{x} - \text{Lower Performance Limit}}{3 \hat{\sigma}}$$

With both the Cp and Cpk indices, larger values are always better:

- When Cpk is greater than 1, the process is in better shape to perform at acceptable levels.

As a rule of thumb, a minimum Cp and Cpk value of 1.33 (4 sigma) is desired by most American companies today, although many companies are seeking Cpk values of 2, 3, 4, and larger before they consider their processes to be capable.

- When the Cpk is equal to 1 (3 sigma), the process may be acceptable *at the moment*, but any shifting of the process or increase in process variation will immediately drive the process to unacceptable performance levels.
- When the Cpk is less than 1, the process is incapable of performing at an acceptable level and should be given immediate attention.



Example

A process is required to be performed on an average of 7 +/- 2 minutes. A control chart was placed on the process and 25 sample groups of 5 items each were collected. It was found that the process was stable and normally distributed with an average time of 7 minutes and an average range of 3.101 minutes.

- $\bar{x} = 7$ minutes
- $\bar{R} = 3.101$ minutes
- USL = 9 minutes (7 + 2)
- LSL = 5 minutes (7 - 2)

- $\hat{\sigma} = \frac{\bar{R}}{d_2} = \frac{3.101}{2.326} = 1.333$ minutes

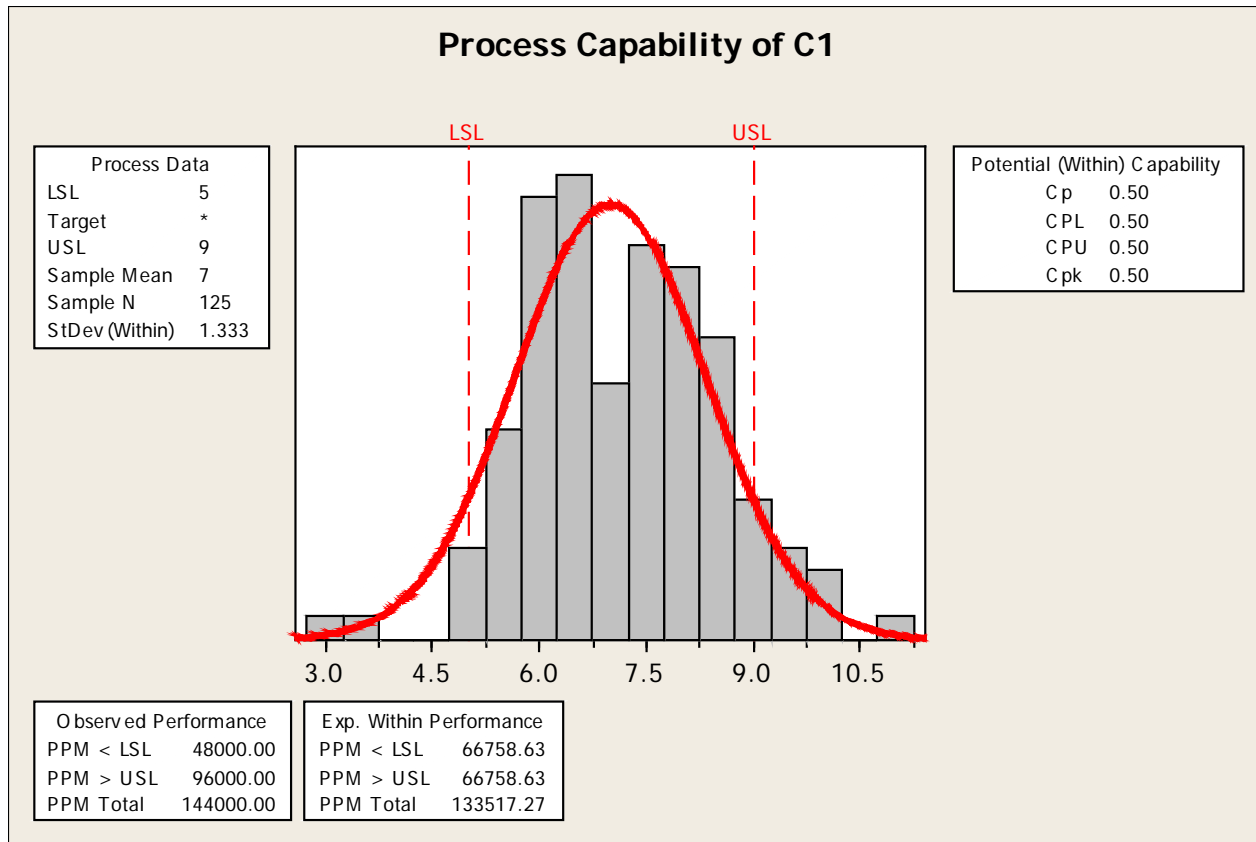
- $C_p = \frac{USL - LSL}{6 \hat{\sigma}} = \frac{9 - 5}{6(1.333)} = 0.50$

- $C_{pk} = \min \text{ of } \frac{\bar{x} - LSL}{3 \hat{\sigma}} = \frac{7 - 5}{3(1.333)} = .50$

or

$$\frac{USL - \bar{x}}{3 \hat{\sigma}} = \frac{9 - 7}{3(1.333)} = .50$$

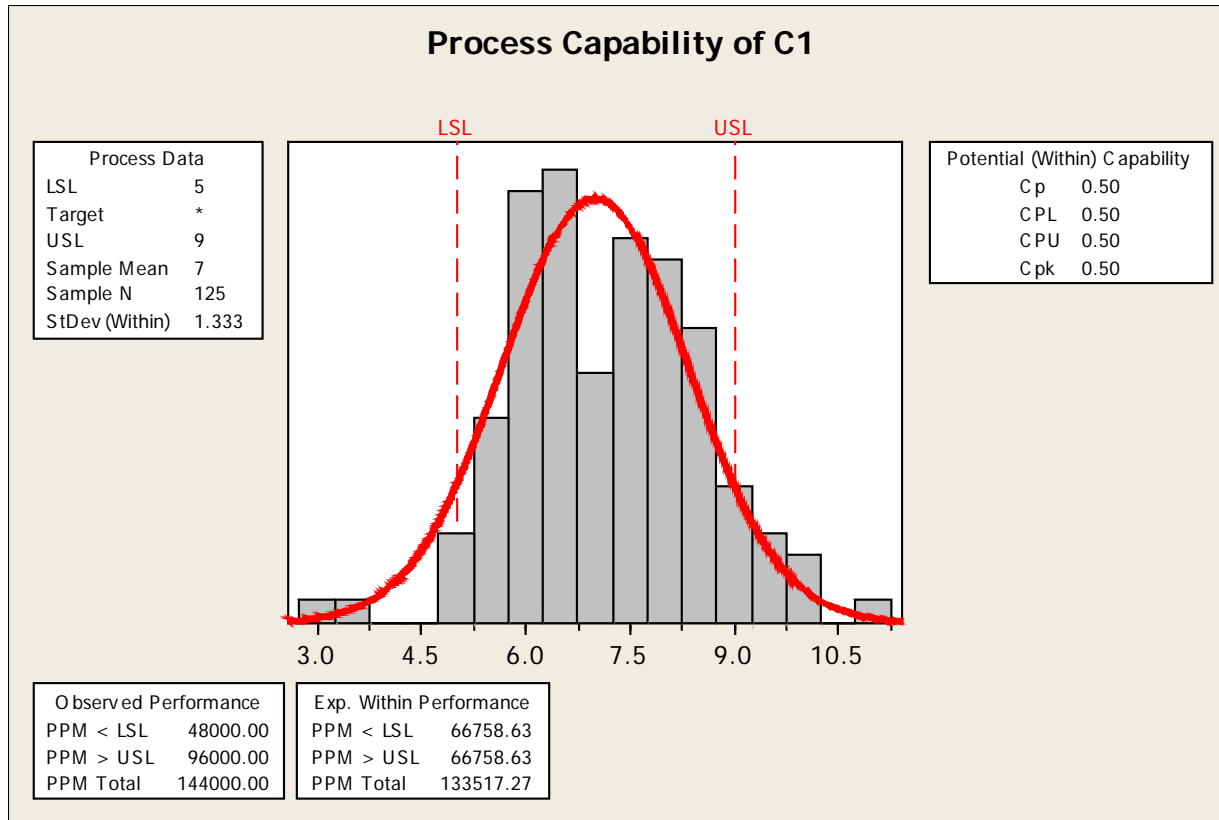
SAMPLE SIZE	d ₂
2	1.128
3	1.693
4	2.059
5	2.326
6	2.534
7	2.704
8	2.847



- A C_p of 0.5 means that the specification limits are half the width of the total predicted process variation.
- A C_{pk} of 0.5 means that half of a Normal curve will fit between the average and either performance limit, so this process is at a 1.5 sigma level.

Special Process Capability Circumstances

What does it mean when the both C_p and C_{pk} are all the same?



- **Equal Cp and Cpk Values:** The Cp and Cpk values will only be equal when a process is centered at the target or nominal value.
- Note that the Cpk value for a given process can never be greater than the Cp value, so if the Cp is unacceptable, the Cpk be unacceptable as well.

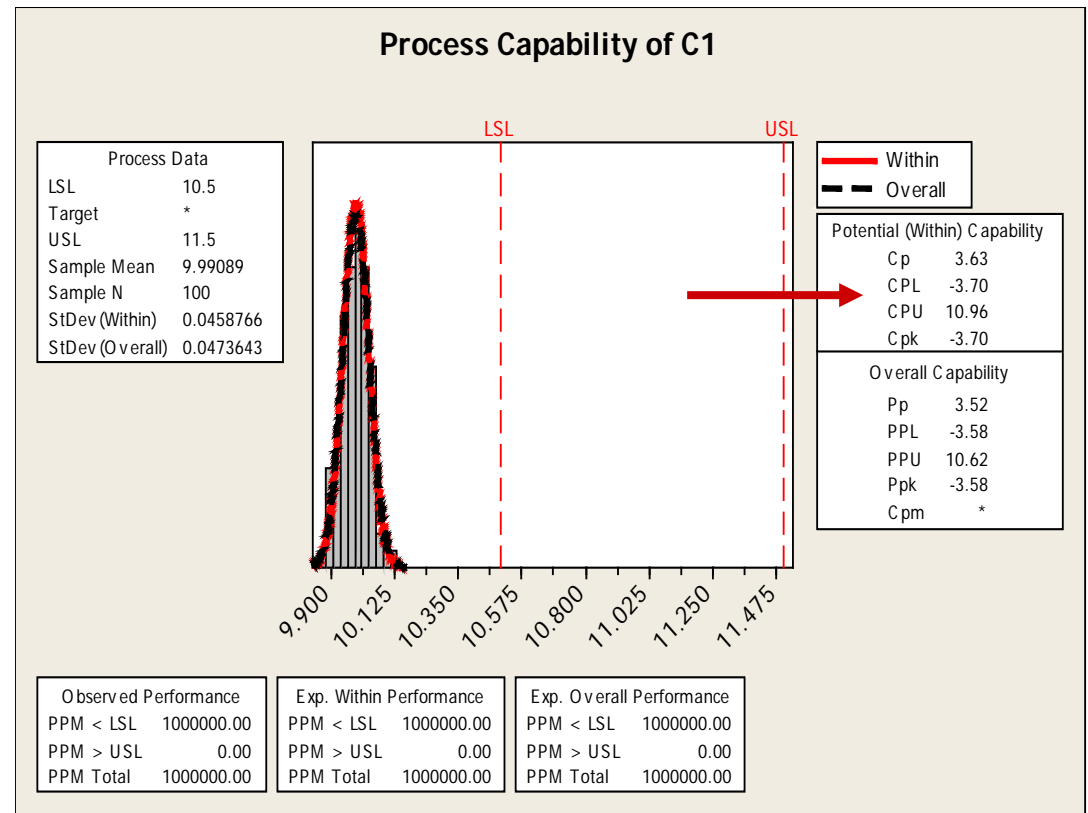
Special Process Capability Circumstances

What does a negative Cpk tell you?

Negative Cpk Values:

Companies will sometimes find their processes centered *outside* of the performance limits. When this occurs, the Cpk calculation still holds true. The only difference is that if the process mean is outside the performance limits (less than the lower performance limit or greater than the upper performance limit), one Cpk value will be a negative number.

The negative value is therefore chosen as the Cpk value since it will always be the lower of the two calculations.



Pp and Ppk

- The Pp and Ppk indices use the overall variation as calculated by the standard deviation for all the data collected across all sample subgroups:

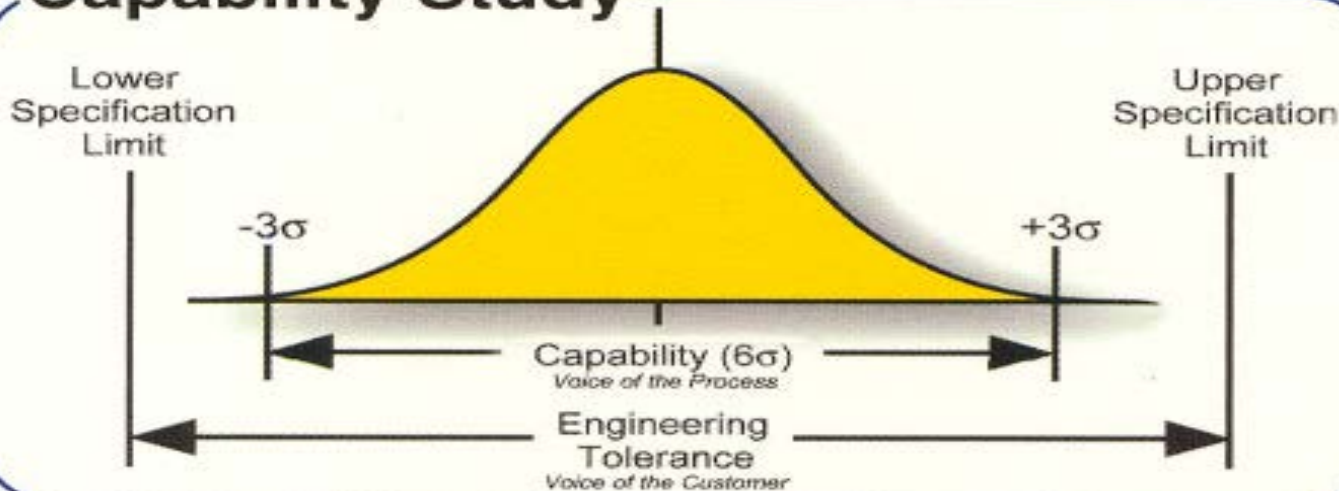
$$s = \sqrt{\frac{\sum(x - \bar{x})^2}{n-1}}$$

- Next two slides I am citing from Douglas Montgomery's text entitled: Introduction to Statistical Quality Control, 6th edition, page 363 and 364.
- "In 1991, the Automotive Industry Action Group (AIAG) was formed and consists of representatives of the "big three" (Ford, General Motors, and Chrysler) and the American Society for Quality Control (now the American Society for Quality). One of their objectives was to standardize the reporting requirements from suppliers and in general of their industry. The AIAG recommends using the process capability indices Cp and Cpk when the process is in control, with the process standard deviation estimated by \bar{R} divided by d_2 .
- When the process is not in control, the AIAG recommends using process performance indices Pp and Ppk."

- “Now it is clear that when the process is normally distributed and in control, P_p is essentially C_p and P_{pk} is essentially C_{pk} , because for a stable process the difference between s and an estimate of s derived from \bar{R} and d_2 is minimal. However, please note that if the process is not in control, the indices P_p and P_{pk} have no meaningful interpretation relative to process capability, because they cannot predict process performance. Furthermore, their statistical properties are not determinable, and so no valid inference can be made regarding their true (or population) values. Also, P_p and P_{pk} provide no motivation or incentive to/the companies that use them to bring their processes into control.”

- When the process is stable, the C_p / C_{pk} indices and the P_p / P_{pk} indices will be very similar.
- If the process is not stable, the use of the C_p / C_{pk} indices is definitely not recommended and the P_p / P_{pk} indices are suspect as well.
- Mathematically how do the formulas for C_{pk} and P_{pk} differ?

Capability Study



Capability Formulas

$$P_{pl} = \frac{\bar{X} - LSL}{3s}$$

$$P_{pu} = \frac{USL - \bar{X}}{3s}$$

$$P_p = \frac{USL - LSL}{6s}$$

$$C_{pl} = \frac{\hat{\mu} - LSL}{3\hat{\sigma}}$$

$$C_{pu} = \frac{USL - \hat{\mu}}{3\hat{\sigma}}$$

$$C_p = \frac{USL - LSL}{6\hat{\sigma}}$$

$$\bar{X} = \frac{\sum X}{n}$$

$$s = \sqrt{\frac{\sum (X_i - \bar{X})^2}{N-1}}$$

$$\hat{\sigma} = \frac{\bar{R}}{d_2}$$

$\hat{\mu}$ = Estimated population mean

Page 303, The Certified Quality Inspector Handbook, H. Fred Walker, Ahmad K. Elshennawy, Bhisham C. Gupta, and Mary McShane-Vaughn, ASQ Press 2009

“Kotz and Lovelace (1998,253) strongly argue against the use of Pp and Ppk.
They have written:

We highly recommend against using these indices when the process is not in statistical control. Under these conditions, the P-numbers are meaningless with regard to process capability, have no tractable statistical properties, and infer nothing about long-term capability of the process. Worse still, they provide no motivation to the user-companies to get their process in control. **The P-numbers are a step backwards in the efforts to properly quantify process capability, and a step towards statistical terrorism in its undiluted form.**

Montgomery (2005, 349) agrees with Kotz and Lovelace. He writes "The process performance indices Pp and Ppk are more than a step backwards. They are a waste of engineering and management effort--they tell you nothing." The authors wholeheartedly agree with Kotz and Lovelace and Montgomery. No one can judge a process when its future behavior is so unpredictable."

- “Unless a process is stable (in control), no index is going to carry useful predictive information about process capability or convey any information about future performance. Instead of imposing the use of meaningless indices, organizations should devote effort to developing and implementing an effective process characterization, control, and improvement plan. This is a much more reasonable and effective approach to process improvement.”
- Douglas C. Montgomery, Introduction to Statistical Quality Control, 6th edition.

- Process Capability Indices are only useful if the process is stable
- Compute your Cpk with your best estimate of the process standard deviation (s)
- Avoid the use of the terms Pp and Ppk
- When using Minitab check you options

Capability Analysis (Normal Distribution): Options



Target (adds Cpm to table):

Use tolerance of $K \times \sigma$ for capability statistics K =

Perform Analysis

- Within subgroup analysis
- Overall analysis

Display

- Parts per million
- Percents

- Capability stats (Cp, Pp)
- Benchmark Z's (σ level)

Include confidence intervals

Confidence level:

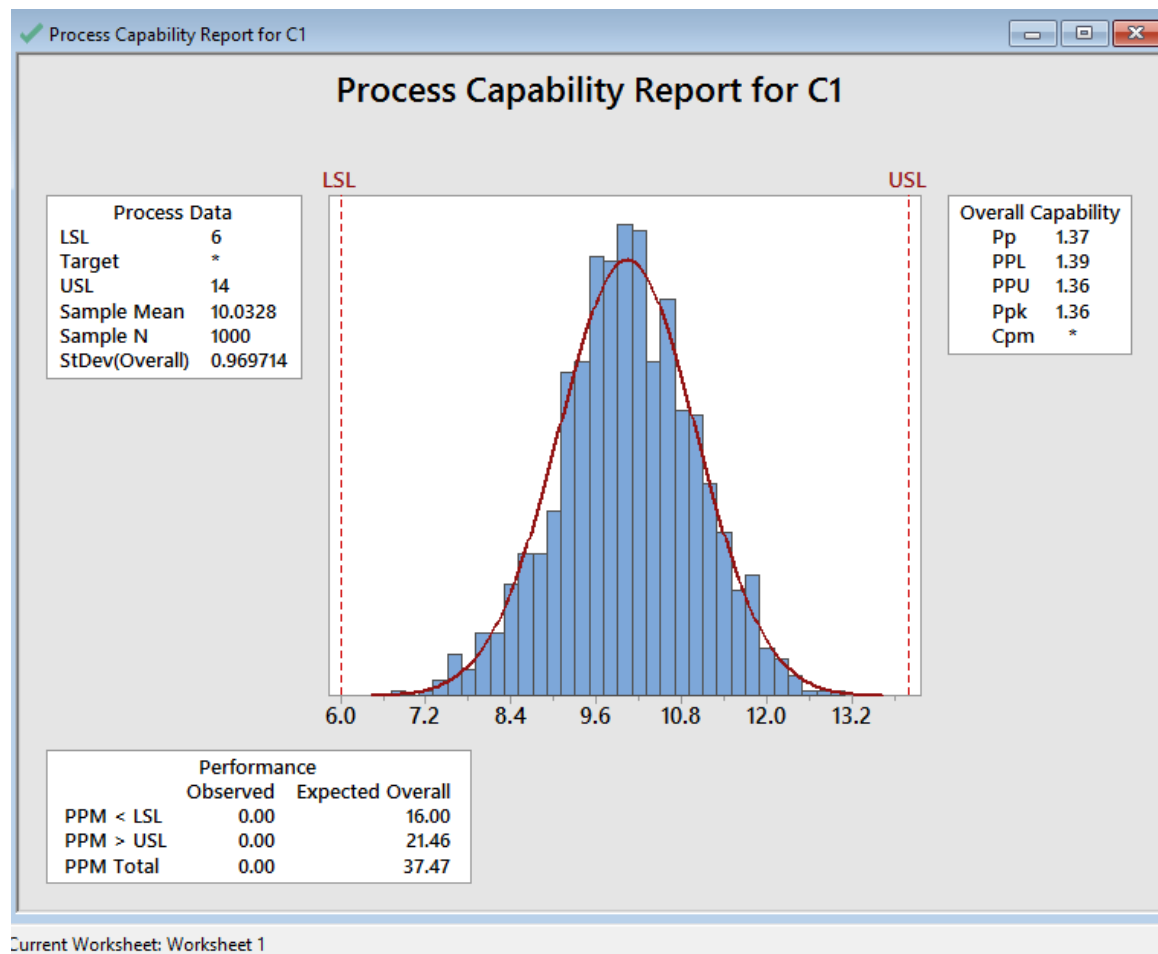
Confidence intervals:

Title:

Help

OK

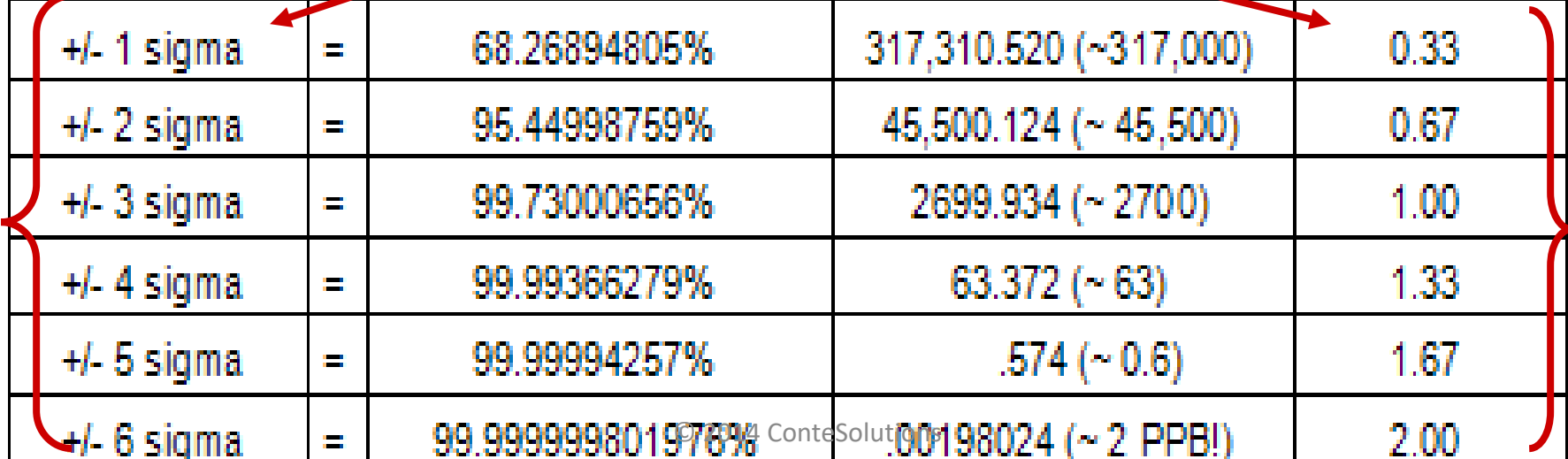
Cancel



Relationship Between Cpk and Sigma Level

<u>SIGMA LEVEL</u> <u>(z SCORE)</u>		<u>PERCENT OF PROCESS</u>		<u>Cp / Cpk</u> (With the process centered)
		<u>WHICH MEETS REQUIREMENTS</u> (With the process centered)	<u>MISTAKES PER MILLION ATTEMPTS (PPM)</u> (With the process centered)	
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+/- 6 sigma	=	99.99999801978%	.00198024 (~ 2 PPB!)	2.00

What do you notice about the relationship between Cp/Cpk and the Sigma level?



The process C_{pk} will always be 1/3 the size of the Sigma level / z score.

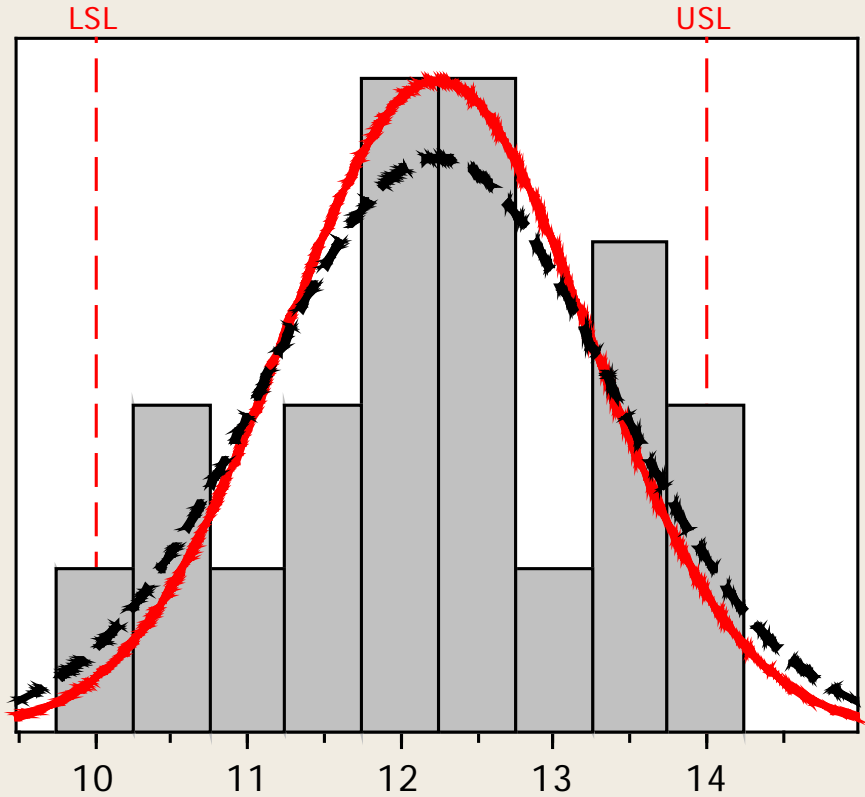
C_{pk} = the smaller of:

$$\frac{\text{Upper Performance Limit} - \bar{x}}{3 \hat{\sigma}} \quad \frac{\bar{x} - \text{Lower Performance Limit}}{3 \hat{\sigma}}$$

$$z = \frac{x - \mu}{\sigma} = \frac{\text{Spec Limit} - \bar{x}}{\sigma}$$

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Z.Bench	1.37
Z.LSL	1.97
Z.USL	1.55
Ppk	0.52
Cpm	*

Observed Performance	
PPM < LSL	0.00
PPM > USL	50000.00
PPM Total	50000.00

Exp. Within Performance	
PPM < LSL	12648.50
PPM > USL	39415.05
PPM Total	52063.54

Exp. Overall Performance	
PPM < LSL	24471.99
PPM > USL	60915.50
PPM Total	85387.49

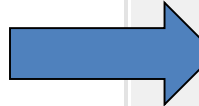
Capability Analysis (Normal Distribution)



C1

Data are arranged as

Single column:



Subgroup size:

(use a constant or an ID column)

Subgroups across rows of:

Transform...

Estimate...

Options...

Storage...

Lower spec:

Boundary

Upper spec:

Boundary

Historical mean:

(optional)

Historical standard deviation:

(optional)

Select

Help

OK

Cancel

Question

- **Dear Sir,**
- **I would like to hear from you what is the best answer to respond if someone asks the question “Why is it Six Sigma? Why not for 7, 8 sigma?”**
- **Hope you can share with me your point of view.**
- **Thanks and best regards,**

The short answer

- The answer to your question is closely related to the topic of Process Capability (pages 363 to 376), a subject that most students in the LSSMBB course have much difficulty. My initial reaction to your question is that "Six Sigma" sounds much better than "4.5 Sigma" which technically was the **Sigma Level** that Motorola engineers were really striving for (see page 368 of the VU Six Sigma Black Belt Online Textbook). To answer the question seriously, I would ask **“What are your current "PPM failures rates”?**
- This question is related to Six Sigma – But asks about Sigma **levels** versus Six Sigma **process improvement methodology**.

The short answer Sigma Level to PPM

- Page 368 VU SS BB Online Textbook
- Sigma level to PPM failures without 1.5 sigma shift
- Sigma Level 1.00 ----- 317,300
- Sigma Level 2.00 ----- 45,500
- Sigma Level 3.00 ----- 2,700
- Sigma Level 4.00 ----- 63
- Sigma Level 4.50 -----3.4
- Sigma Level 5.00 ----- 0.6

- Sigma Level 6.00 ----- 0.002

My more complicated answer

- **How many processes have you defined, collected enough data to estimate a process average and process standard deviation, and have established specification limits?**
- **0, 1, 3, 20, 300, 1000+ ?**
- **What is your definition of “defined”?**
- **How much data was collected for your process average?**
- **How current is the estimate of the process average?**
- **How did you determine your specifications?**
- **What is your average sigma level?**
- **What is your average PPM failure rates?**

My more complicated answer, cont'

Defined Process	Repetitions per month	Process Average	Process std deviation	Nearest Spec	Sigma Level	PPM Failure Rates
Sandwich Creation	900	1.45	0.25	< 1.50	0.20	420,700
Daily Setup	30	22.0	1.4	< 30	5.71	<1
Daily Shutdown	30	35.6	4.9	< 30	-1.14	873,500
Average Weighted ave					1.59 0.33	431,000 421,700

$\text{Sigma level} = (\text{nearest Spec limit} - \text{process mean}) / \text{process standard deviation}$

So again I ask?

- **How many processes have you defined, collected enough data to estimate a process average, and process standard deviation, and have established specification limits?**