Design of Experiments

Notebook Tab 6

Pages 269 to 329

H. Design and Analysis of Experiments

1. **Terminology**
   Define terms such as dependent and independent variables, factors, levels, response, treatment, error, and replication. (Understand)

2. **Planning and organizing experiments**
   Define, describe, and apply the basic elements of designed experiments, including determining the experiment objective, selecting factors, responses, and measurement methods, choosing the appropriate design, etc. (Analyze)

3. **Design principles**
   Define and apply the principles of power and sample size, balance, replication, order, efficiency, randomization, blocking, interaction, and confounding. (Apply)

4. **One-factor experiments**
   Construct one-factor experiments such as completely randomized, randomized block, and Latin square designs, and use computational and graphical methods to analyze the significance of results. (Analyze)

5. **Full-factorial experiments**
   Construct full-factorial designs and use computational and graphical methods to analyze the significance of results. (Analyze)

6. **Two-level fractional factorial experiments**
   Construct two-level fractional factorial designs (including Taguchi designs) and apply computational and graphical methods to analyze the significance of results. (Analyze)

The lead into DOE

- Normal, Chi-square, F probability distributions
- Statistical Significance, critical region
- Test of Hypothesis

A designed experiment is:

a test or series of tests in which purposeful changes are made to the input variables of a process or system so that the experimenter may observe and identify the magnitude of and reasons for changes in the output response.

History of Designed Experiments

- Between 1922 and 1926, Sir Ronald A. Fisher, working in England with universities and agricultural experiment stations, developed the basic idea of designed experiments.

- During the 1930's, Fisher extended the application of designed experiments to industrial problems.

- Fisher continued to develop and apply statistical methods until his death in 1962.

Some Applications of DOE

- Improve process yields.
- Reduce variability.
- Reduce development time.
- Optimize products and processes.
- Evaluate material alternatives.
- Ascertain key performance variables.

Drawbacks to "One Factor at a Time" Experimentation

- It may take numerous "best guesses" before an acceptable result is obtained, and by this point the experiment may actually exceed the size and cost of a more formal one.
- Even when an acceptable solution is found, the experimenter has no guarantee that it is the optimal one without further testing.
- There is no ability to test for interaction.

Key DOE Terms and Concepts

- **HYPOTHESIS**: An assertion or belief about the effect of a particular treatment on the process under study. A *test of the hypothesis* is therefore a test of the validity of this assertion, and is conducted by analyzing the sample data obtained during the experiment.

- **EXPERIMENT**: A test or series of tests in which purposeful changes are made to the input (independent variables / factors) of a process or system so that the reasons for changes in the output (response / dependent variables) may be observed and evaluated.

- **DESIGN OF THE EXPERIMENT**: The plan for conducting the experiment (factors tested, settings at which they are tested, number of units tested, analysis method, etc.)

- **RESPONSE (OR DEPENDENT) VARIABLE**: The output which is being measured / evaluated to determine the effect of the experimental manipulation; the output characteristic of interest.

- **EXPERIMENTAL UNIT**: The object upon which the response variable is measured; the sample item.

- **FACTORS (OR INDEPENDENT) VARIABLES:** The process conditions/parameters theorized to affect the response variable and which are changed during the experiment to determine the resulting impact on the response variable.

- **LEVELS**: The values or intensity settings of the factors / independent variables in the experiment.

- **TREATMENT / TREATMENT COMBINATION**: A particular combination of the levels of the factors in the experiment.

- **REPETITION**: The number of experimental units tested under each experimental run; also known as *sample size*.

- **REPLICATION**: A completely new run (including new set-up) of a particular treatment combination.

- **MAIN EFFECTS**: When there is an optimum level / setting for a tested factor regardless of the levels at which other tested factors are set.

- **INTERACTION**: When the optimum level / setting for a tested factor depends upon the level at which other factors are set.

Example of a Full Factorial Experiment

Consider this experiment in a machine shop to improve the smoothness of the parts produced. They believe that the Temperature of the part, the Speed of the lathe, and the Pressure of the tool against the stop affect smoothness. (With smoothness measures, 0 is optimum.)

Factor A = Temperature Levels: 40° F, 60° F

Factor B = Lathe Speed Levels: 400 RPM, 650 RPM

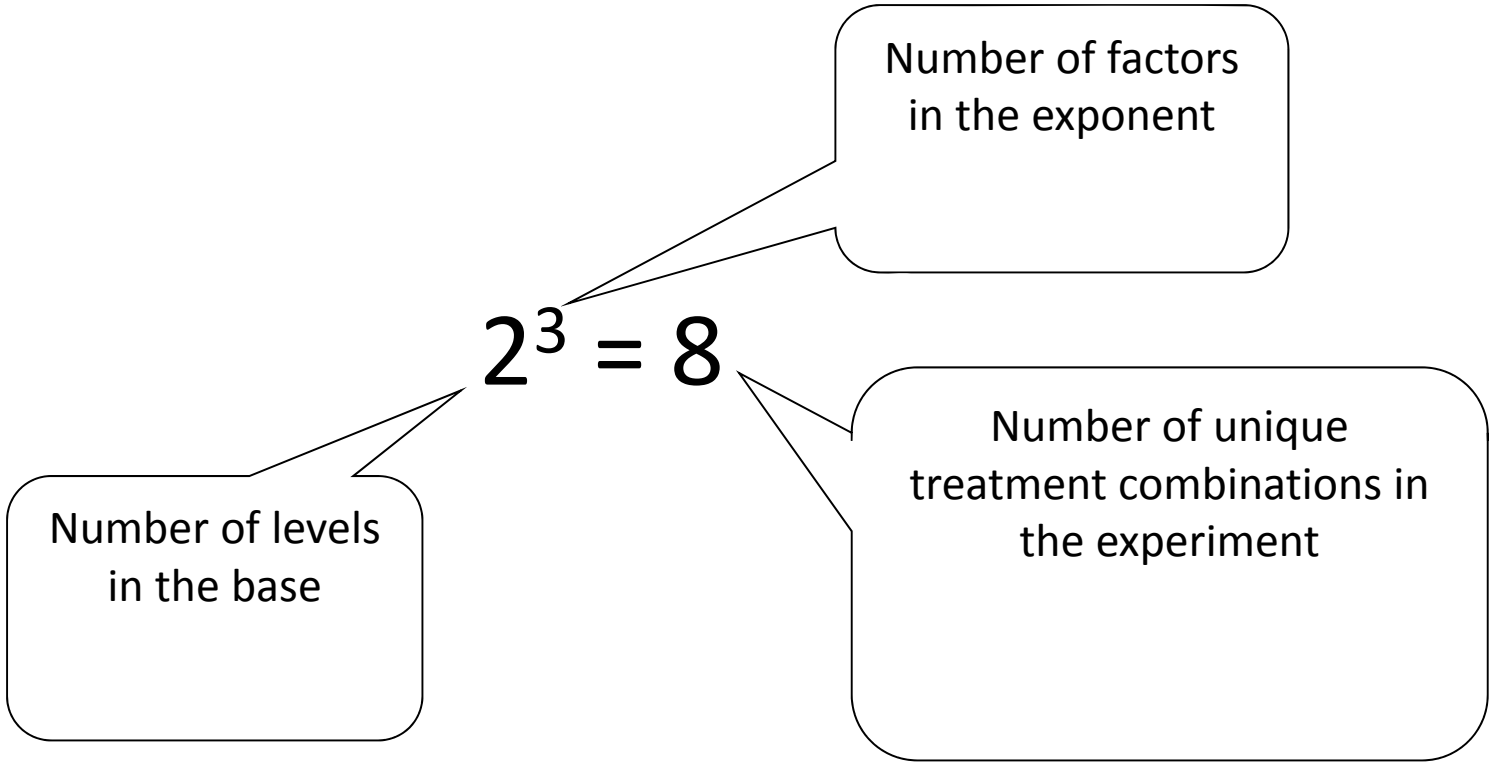
Factor C = Pressure of the tool against the stop

Levels: 50 PSI, 70 PSI

Result Y = Smoothness of the material (response variable where "0" is optimum smoothness)

Experimental Unit: Part # 867-A

This is a 3 factor, 2 level design with $2^3 = 8$ unique treatment combinations.



Available Factorial Designs (with Resolution)

	Factors													
Run	2	3	4	5	6	7	8	9	10	11	12	13	14	15
4	Full	III												
8		Full	IV	III	III	III								
16			Full	V	IV	IV	IV	III	III	III	III	III	III	III
32				Full	VI	IV	IV	IV	IV	IV	IV	IV	IV	IV
64					Full	VII	V	IV	IV	IV	IV	IV	IV	IV
128						Full	VIII	VI	V	V	IV	IV	IV	IV

TREATMENT COMBINATIONS:

Temp	Speed	Pressure
40	400	50
60	400	50
40	650	50
60	650	50
40	400	70
60	400	70
40	650	70
60	650	70

Worksheet 1 ***

↓	C1	C2	C3	C4
	Temp	Speed	Pressure	
1	40	400	50	5
2	60	400	50	2
3	40	650	50	1
4	60	650	50	2
5	40	400	70	5
6	60	400	70	2
7	40	650	70	4
8	60	650	70	2
9	40	400	50	4
10	60	400	50	4
11	40	650	50	2
12	60	650	50	1
13	40	400	70	3
14	60	400	70	3
15	40	650	70	5
16	60	650	70	3

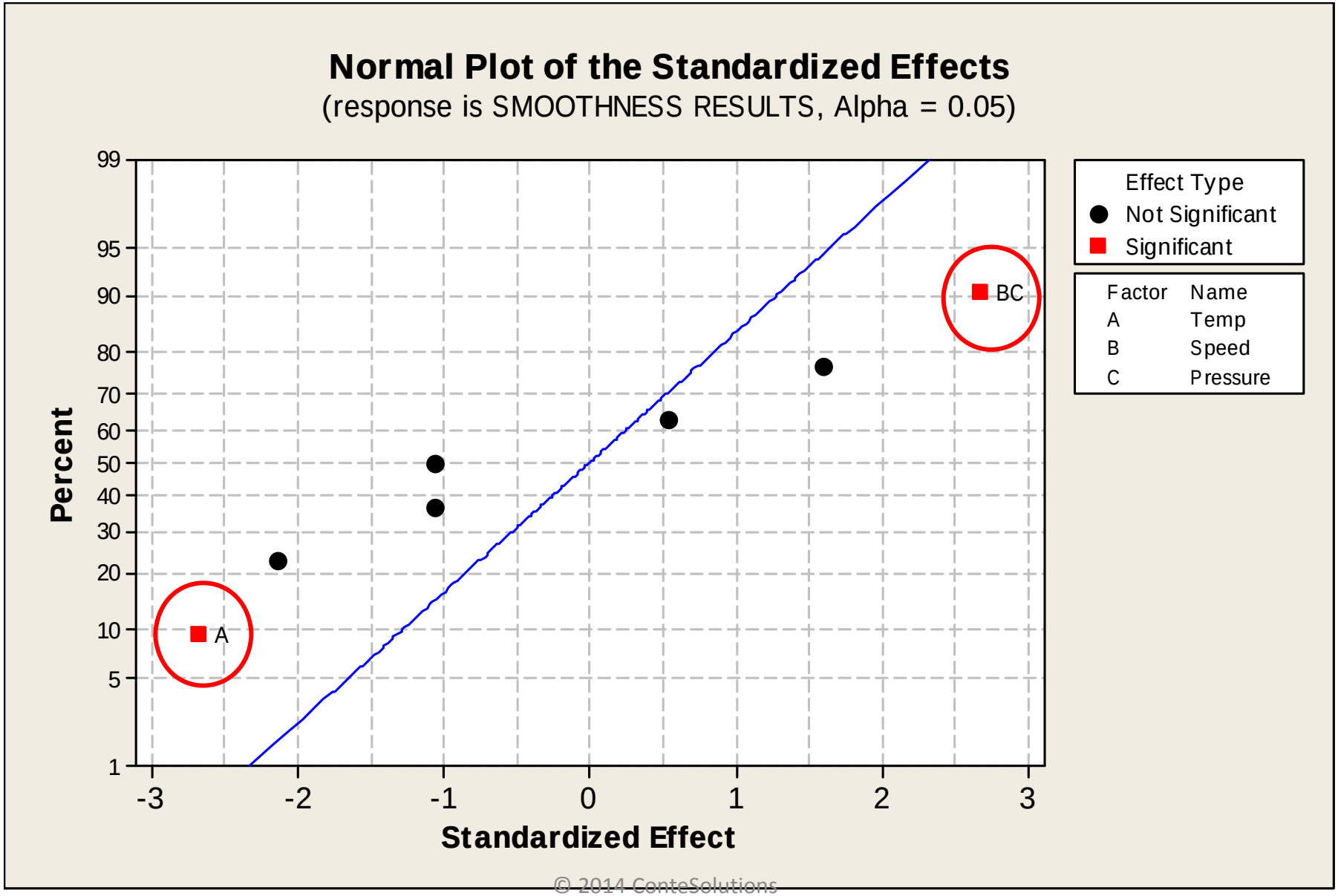
ANOVA: SMOOTHNESS RESULTS versus Temp, Speed, Pressure

Factor	Type	Levels	Values
Temp	fixed	2	40, 60
Speed	fixed	2	400, 650
Pressure	fixed	2	50, 70

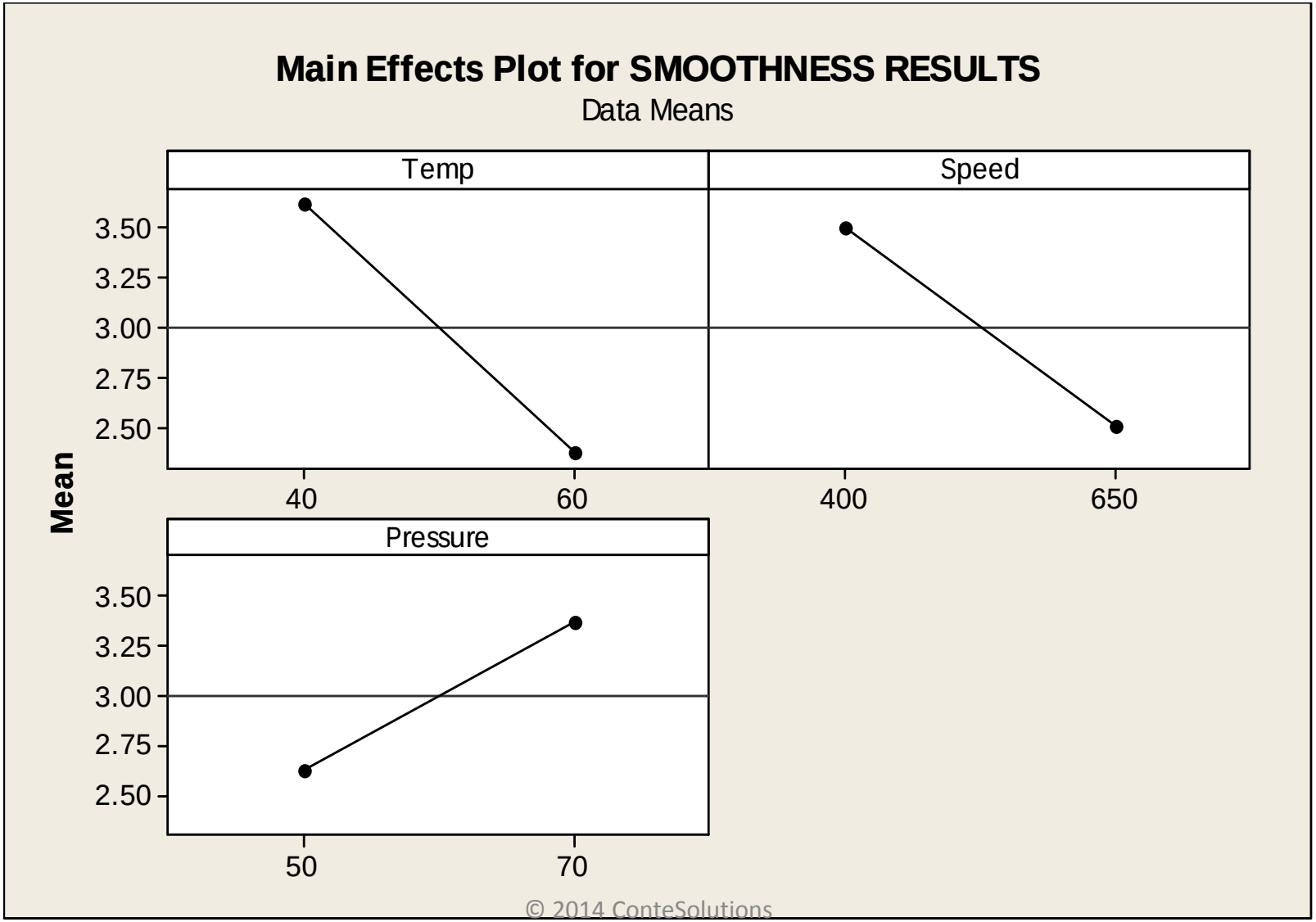
Analysis of Variance for SMOOTHNESS RESULTS

Source	DF	SS	MS	F	P
Temp	1	6.2500	6.2500	7.14	0.028
Speed	1	4.0000	4.0000	4.57	0.065
Pressure	1	2.2500	2.2500	2.57	0.147
Temp*Speed	1	0.2500	0.2500	0.29	0.608
Temp*Pressure	1	1.0000	1.0000	1.14	0.316
Speed*Pressure	1	6.2500	6.2500	7.14	0.028
Temp*Speed*Pressure	1	1.0000	1.0000	1.14	0.316
Error	8	7.0000	0.8750		
Total	15	28.0000			

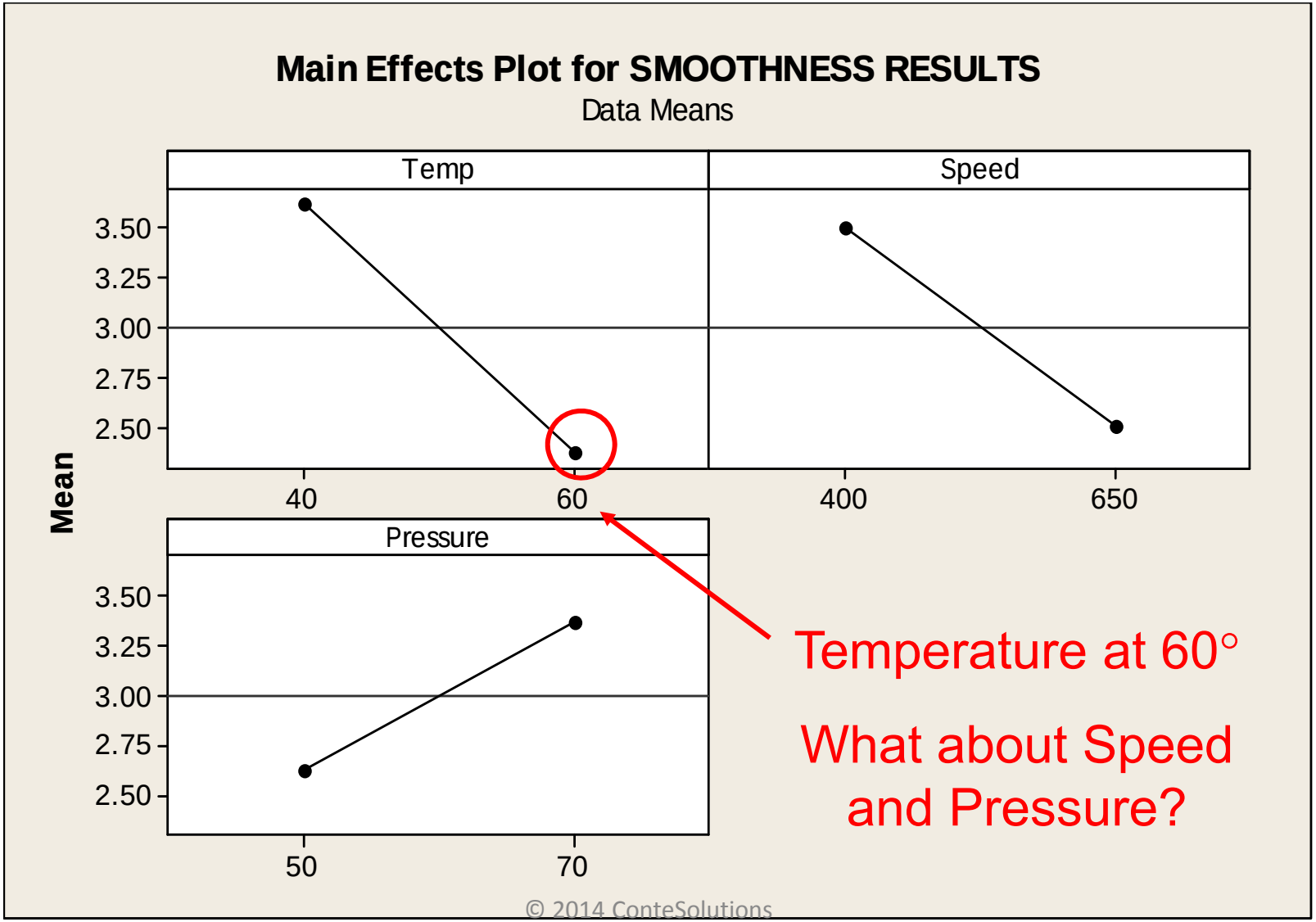
S = 0.935414 R-Sq = 75.00% R-Sq(adj) = 53.13%



What are the optimum settings?

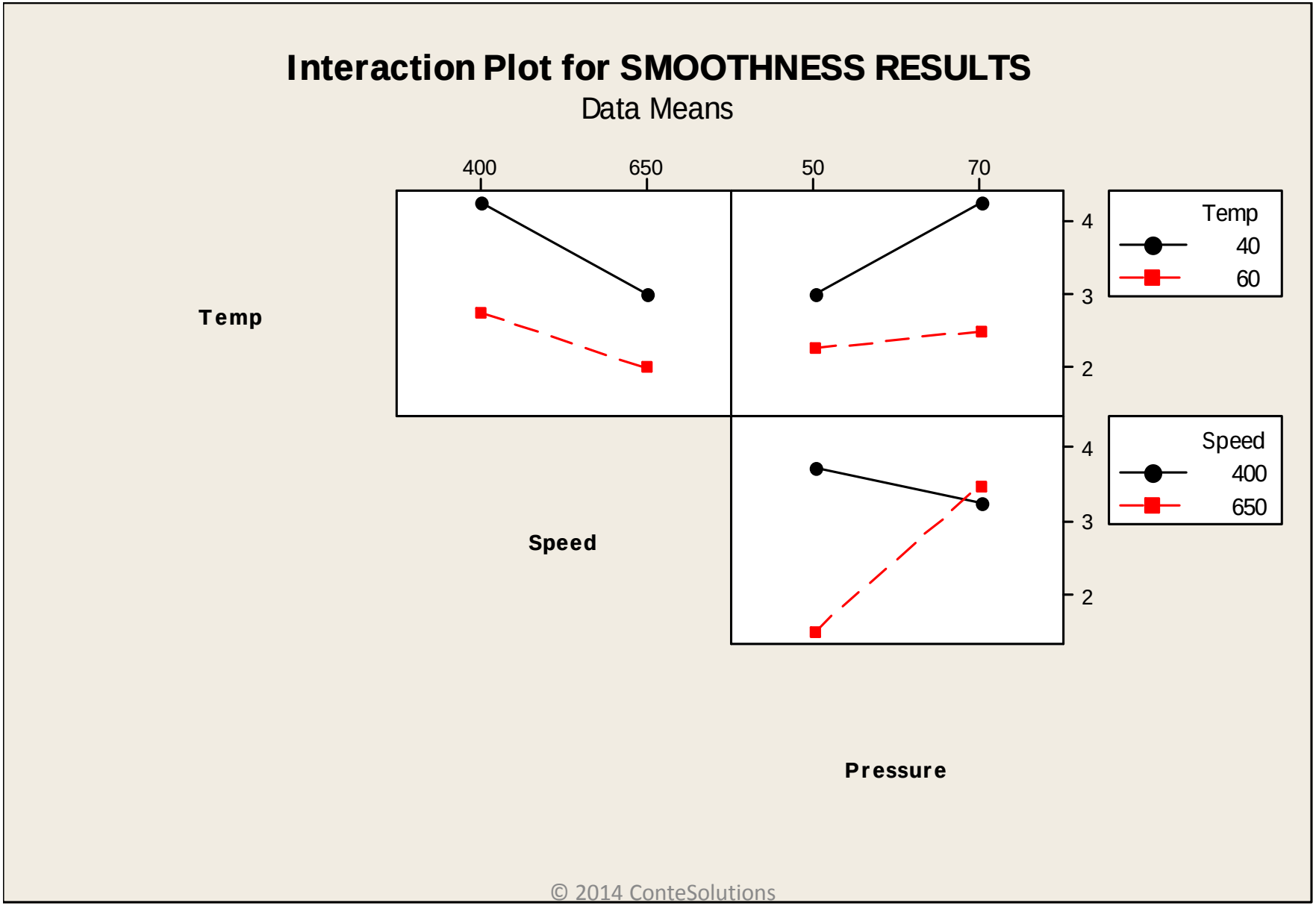


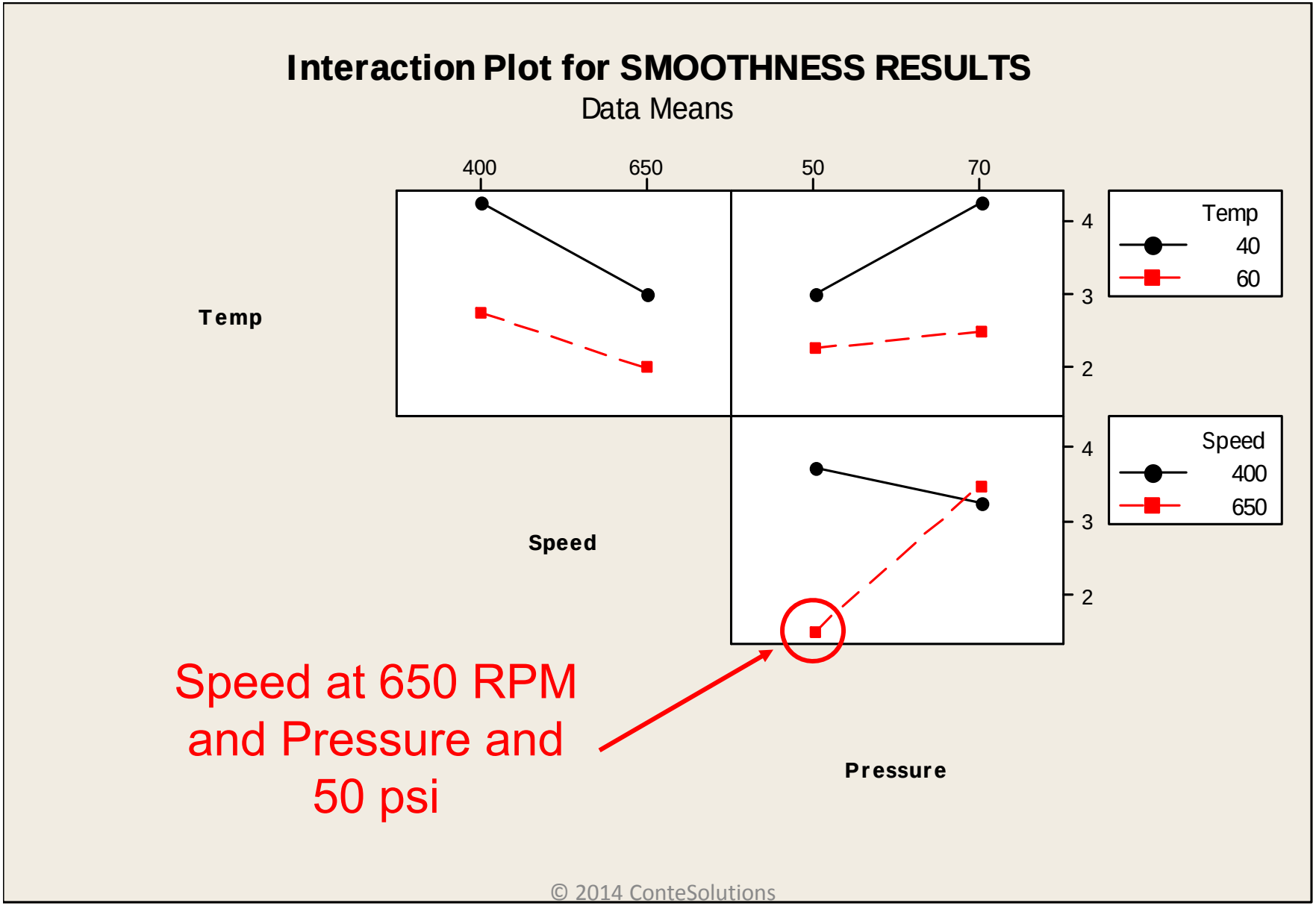
What are the optimum settings?



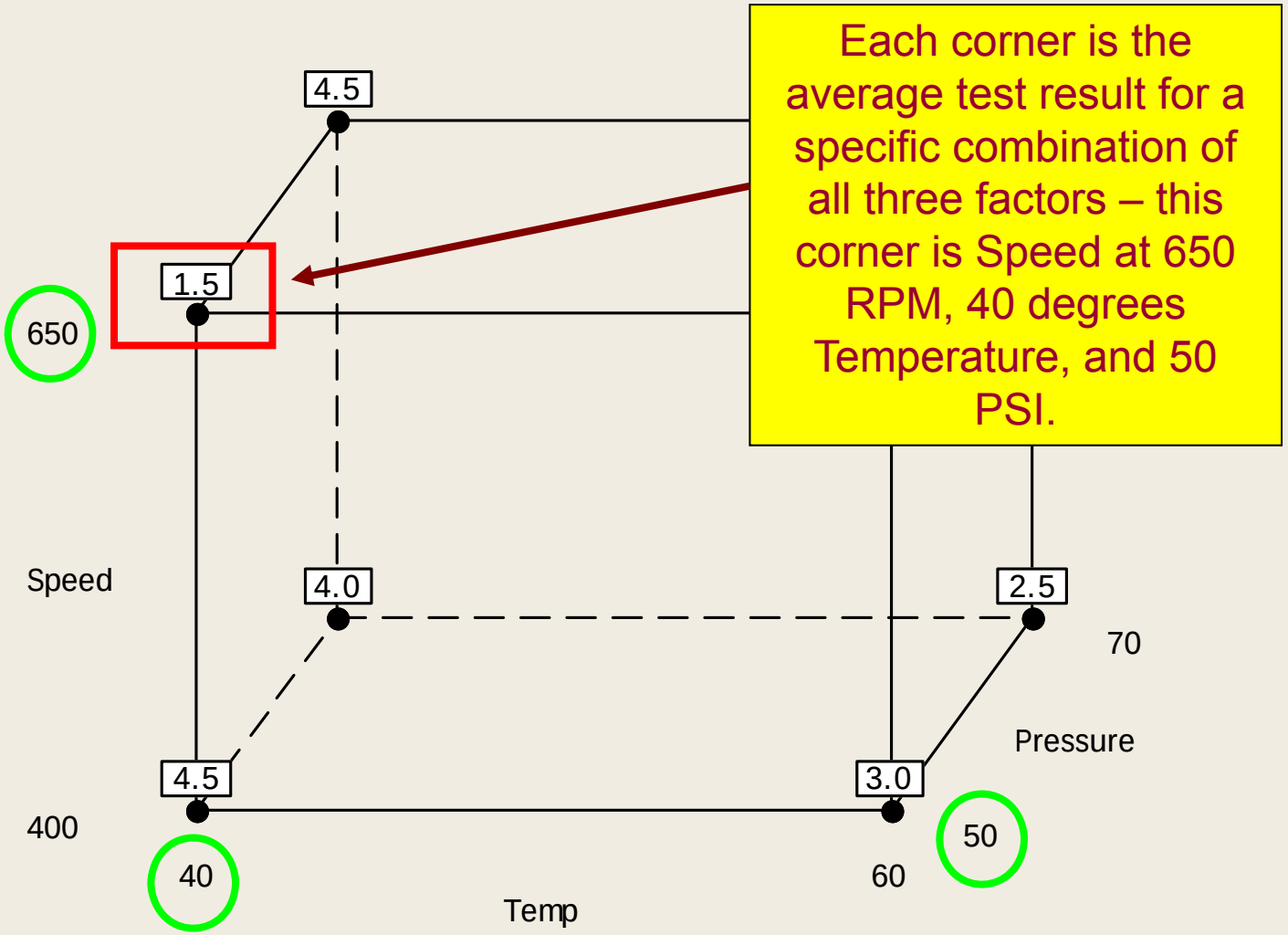
Temperature at 60°

What about Speed and Pressure?

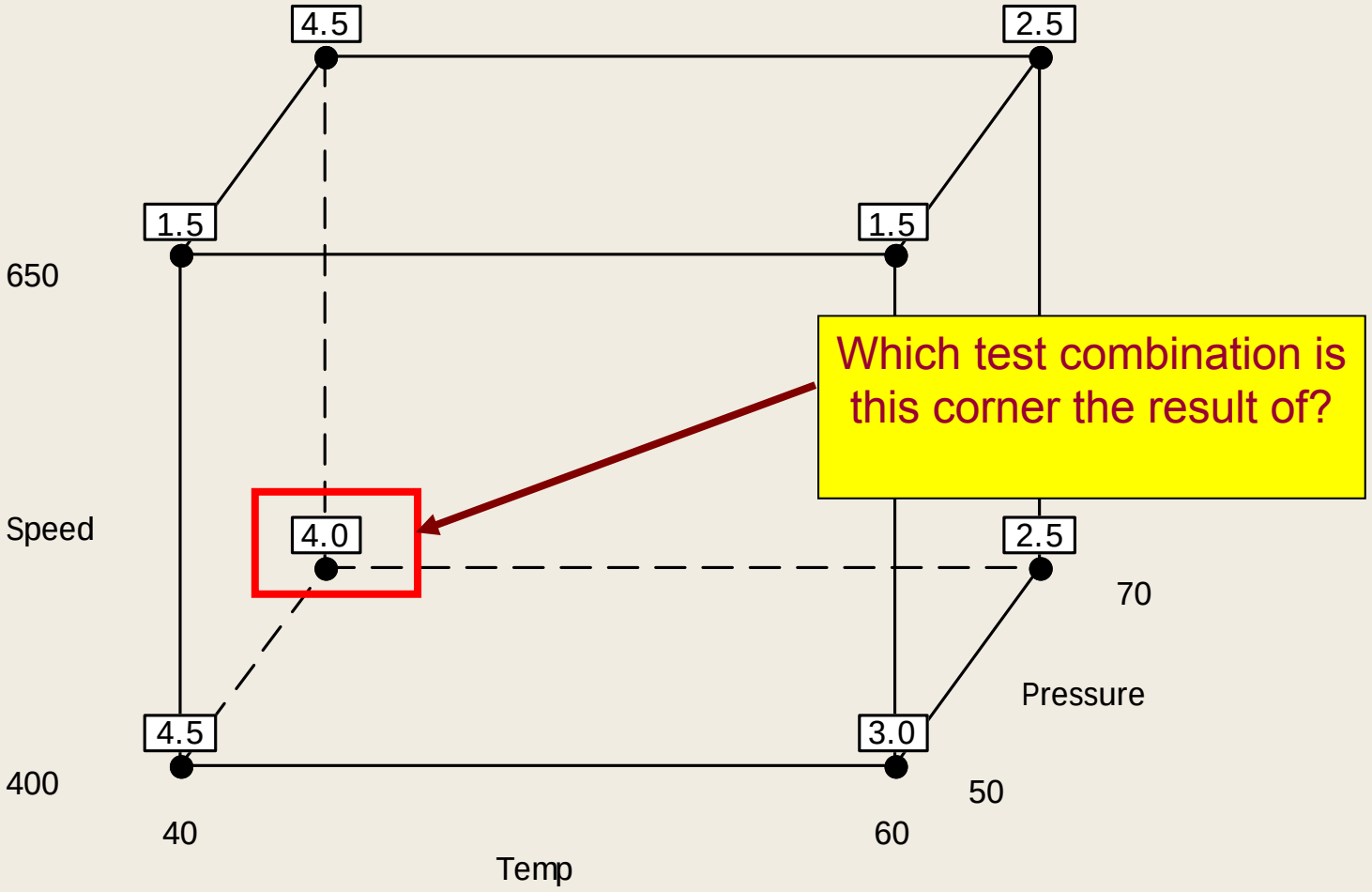




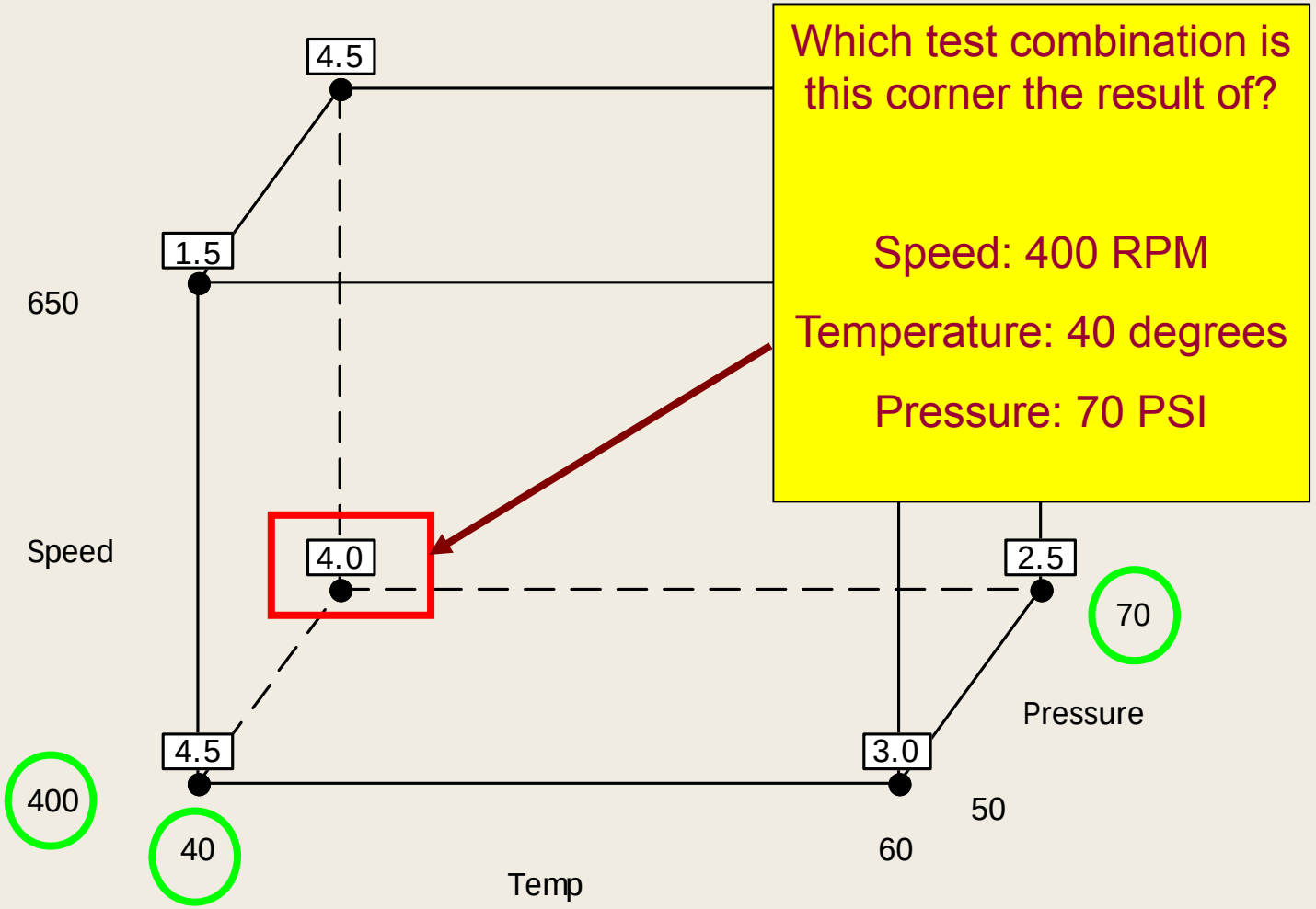
Cube Plot (data means) for SMOOTHNESS RESULTS



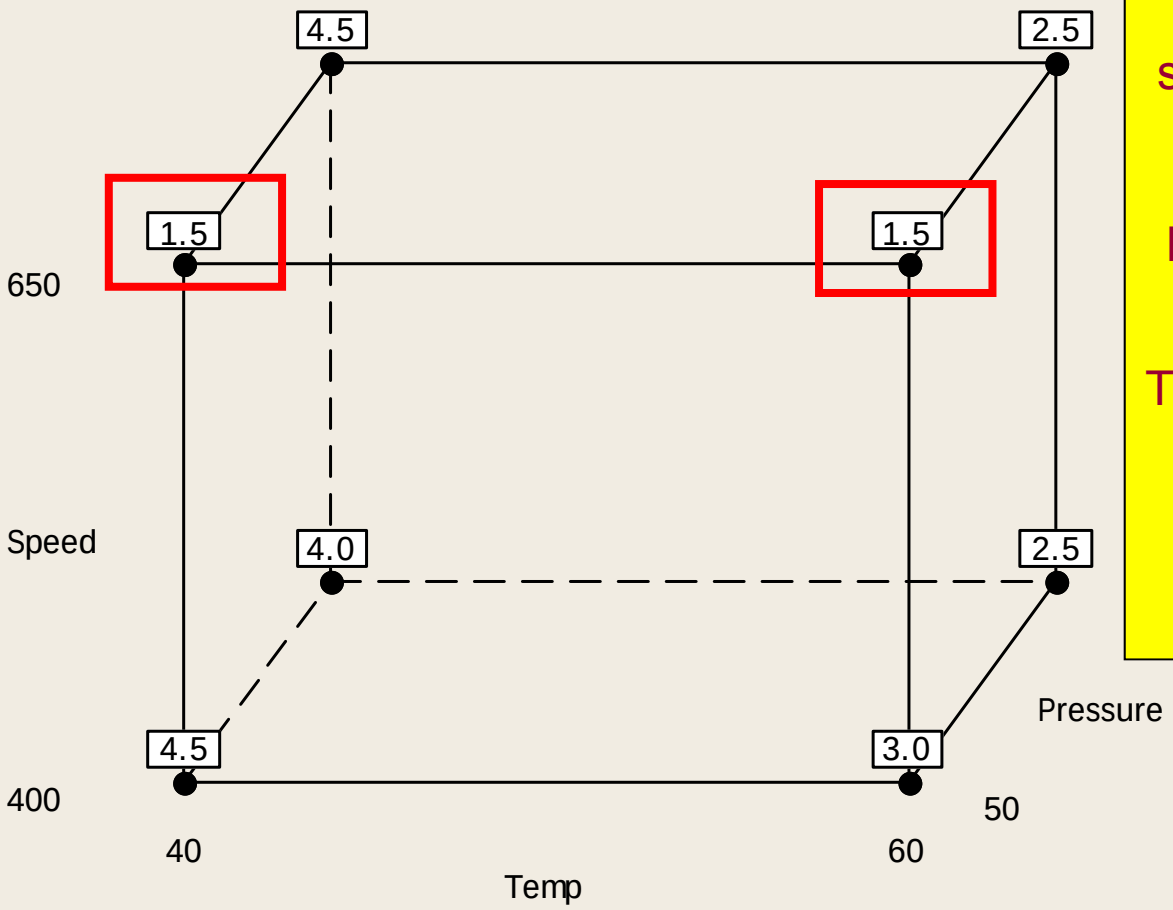
Cube Plot (data means) for SMOOTHNESS RESULTS



Cube Plot (data means) for SMOOTHNESS RESULTS



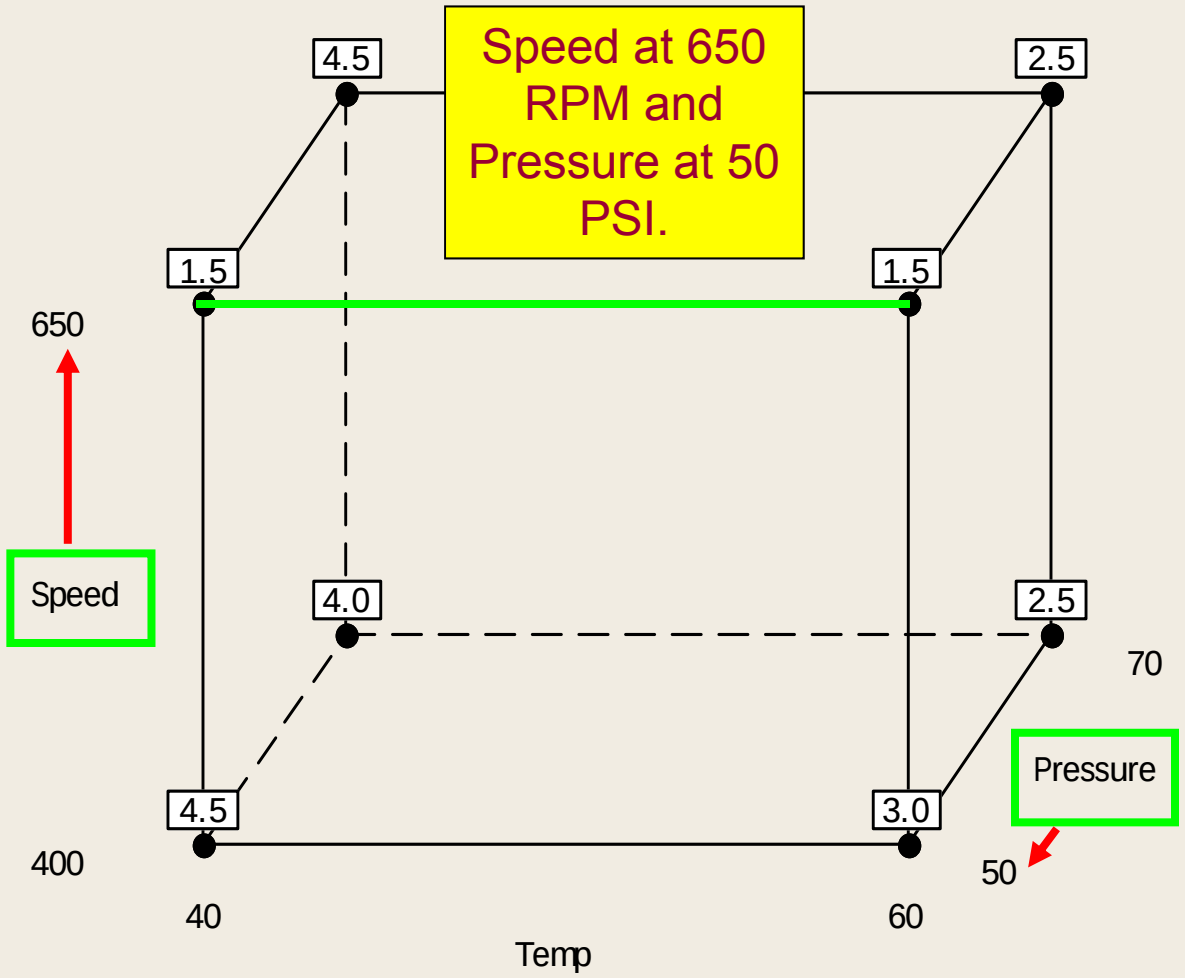
Cube Plot (data means) for SMOOTHNESS RESULTS



If there is a significant 3-factor interaction, one corner will be better than all the others.

This is not the case here since the 3-fctor interaction was not found to be significant.

Cube Plot (data means) for SMOOTHNESS RESULTS



Another example

- 10 steps
- Datasheet
- Design Order, random order generator
- Main Effects Plot
- Analyze Factorial Design
- ANOVA
- Four factor example, too many interactions tested

10 step DOE

GENERATING A 2^4 FULL FACTORIAL DESIGNED EXPERIMENT:

A WAVE SOLDER EXAMPLE

The steps and an example for creating a 2^4 full factorial experiment are as follows:

1. Select the four factors to be investigated based on the engineering judgment, previous experiments, etc. Designate these factors A, B, C, and D.

   Experimenters wish to determine the levels at which to set the belt speed, pre-heat temperature, and angle of incline of a wave solder machine in order to minimize the number of solder defects. They also want to know the effects of the type of solder flux used. They further believe that these four factors are not independent of each other, but work in combination in some way (interact). A factorial designed experiment is therefore necessary to fully test these four factors as well as any interaction effects that might exist. They arbitrarily designate the flux as factor A, the belt speed as factor B, the angle of incline as factor C, and the pre-heat temperature as factor D.

2. Determine two levels for each factor. The first level, labeled "-" or "0" is usually, but not always, the current level for that factor. If the first level is the level currently used, the second level, labeled "+" or "1" is assumed to be the better level, and is, therefore, being tested to verify this.

LETTER	FACTOR	"-" LEVEL (Current)	"+" LEVEL (Proposed)
A	Flux	A19	A880
B	Belt Speed	4 ft./min.	6 ft./min.
C	Angle of Incline	5°	7°
D	Pre-Heat Temp.	160° F	220° F

3. Create a table (known as a matrix) showing the sixteen combinations (treatments) being tested.

		FLUX A- (A19)		FLUX A+ (A880)	
		SPEED B- (4'/min)	SPEED B+ (6'/min)	SPEED B- (4'/min)	SPEED B+ (6'/min)
ANGLE C- (5°)	PRE-HEAT D- (160° F)				
	PRE-HEAT D+ (220° F)				
ANGLE C+ (7°)	PRE-HEAT D- (160° F)				
	PRE-HEAT D+ (220° F)				

4. Randomize the sequence in which the combinations are to be tested. The order selected is shown for each treatment combination.

		FLUX A- (A19)		FLUX A+ (A880)	
		SPEED B- (4"/min)	SPEED B+ (6"/min)	SPEED B- (4"/min)	SPEED B+ (6"/min)
ANGLE C- (5°)	PRE-HEAT D- (160° F)	#6	#3	#2	#11
	PRE-HEAT D+ (220° F)	#10	#7	#12	#14
ANGLE C+ (7°)	PRE-HEAT D- (160° F)	#5	#1	#15	#8
	PRE-HEAT D+ (220° F)	#16	#9	#13	#4

5. Conduct an experiment with each combination, in the sequence identified by the randomization process. Record the results (output characteristic) in the appropriate cell in the table. In this example, the output characteristic measured is the number of solder defects found in the entire sample.

		FLUX A- (A19)		**FLUX A+ (A880)**	
		SPEED B- (4'/min)	**SPEED B+ (6'/min)**	**SPEED B- (4'/min)**	**SPEED B+ (6'/min)**
ANGLE C- (5°)	**PRE-HEAT D- (160° F)**	#6 21	#3 14	#2 104	#11 8
	PRE-HEAT D+ (220° F)	#10 17	#7 64	#12 1	#14 0
ANGLE C+ (7°)	PRE-HEAT **D- (160° F)**	#6 4	#1 43	#15 44	#8 3
	PRE-HEAT D+ (220° F)	#16 32	#9 14	#13 10	#4 0

6. Replicate the experiment by repeating Steps 4 and 5, with a *different* random order for the tests. Since all experiments have residual error in their results, it is always recommended that an experiment be replicated (repeated) at least once.

		FLUX A- (A19)		**FLUX A+ (A880)**	
		SPEED B- (4'/min)	**SPEED B+ (6'/min)**	**SPEED B- (4'/min)**	**SPEED B+ (6'/min)**
ANGLE C- (5°)		#1	#3	#2	#4
	PRE-HEAT	21	14	104	8
	D- (160° F)	17	16	112	8
		#9	#11	#10	#12
	PRE-HEAT	17	64	1	0
	D+ (220° F)	15	58	1	0
ANGLE C+ (7°)		#5	#7	#6	#8
		4	43	44	3
	PRE-HEAT	4	47	38	3
	D- (160° F)				
		#13	#15	#14	#16
		32	14	10	0
	PRE-HEAT	34	12	10	0
	D+ (220° F)				

10. All factors are usually tested for their effects alone (known as **Main Effects**), as well as in combination with all other factors (**Interactions**). Interactions may be tested in combinations of two or more, but higher-order interactions (3-way or more) are fairly uncommon and can often safely be ignored. This is especially useful when the ANOVA is being performed manually, since the calculation of the sums of squares for higher-order interactions is quite complex. When computers are being used for the analysis, it is recommended that all higher order interactions be tested. The sums of squares for the total experiment, every factor, every interaction of interest, and the experimental error must all be calculated to complete the analysis. Interaction is noted as if the factors are being multiplied. For example, the interaction between factors A and B is noted by "A x B" or "AB".

In this example, the Main Effects and Interactions are:

MAIN EFFECTS:

A
B
C
D

2-FACTOR INTERACTIONS:

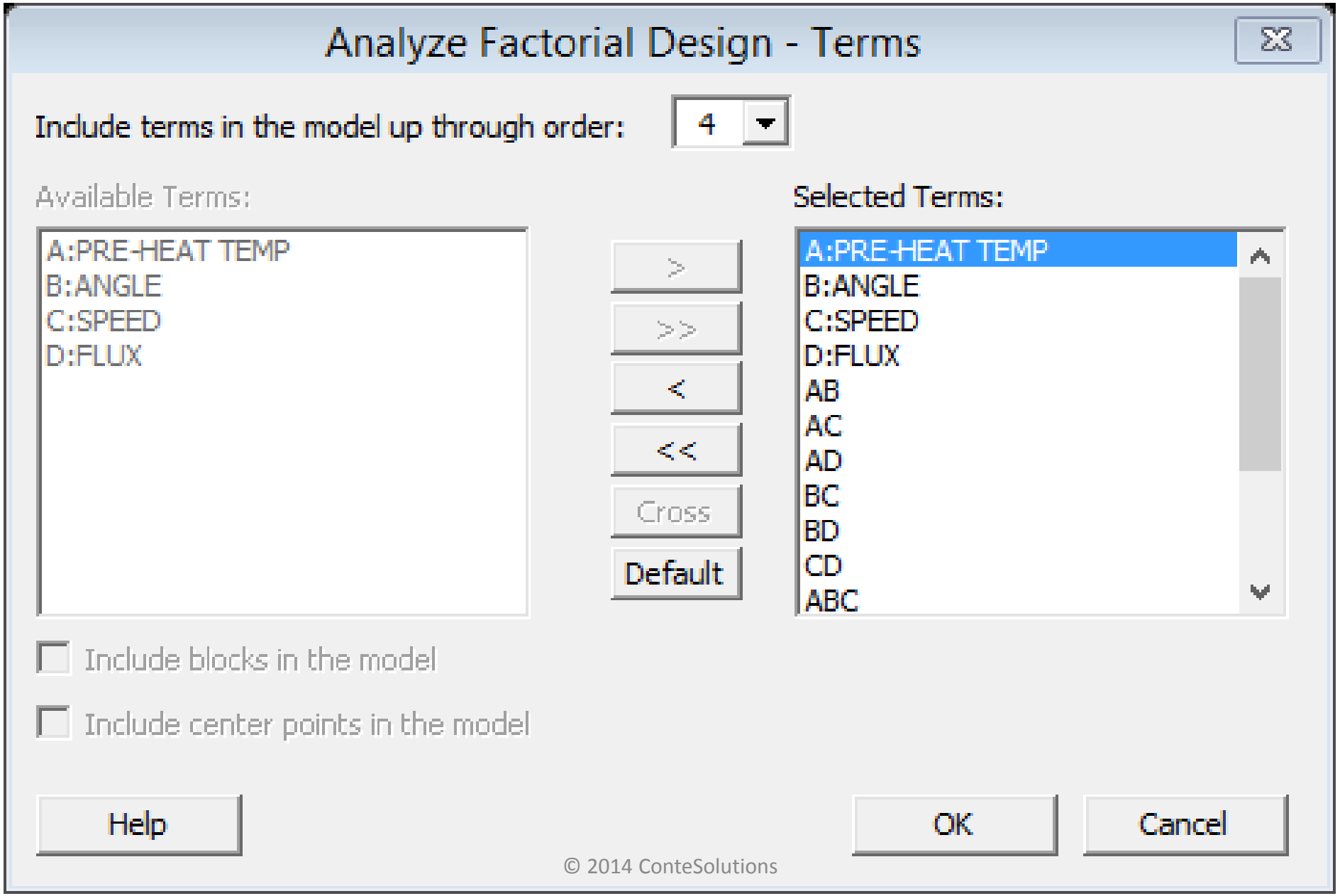
AB
AC
AD
BC
BD
CD

3-FACTOR INTERACTIONS:

ABC
ABD
ACD
BCD

4-FACTOR INTERACTIONS:

ABCD


Analyze Factorial Design - Terms
Include terms in the model up through order:
4
Available Terms:
A:PRE-HEAT TEMP
B:ANGLE
C:SPEED
D:FLUX
>
>>
<
<<
Cross
Default
Selected Terms:
A:PRE-HEAT TEMP
B:ANGLE
C:SPEED
D:FLUX
AB
AC
AD
BC
BD
CD
ABC
Include blocks in the model
Include center points in the model
Help
OK
Cancel
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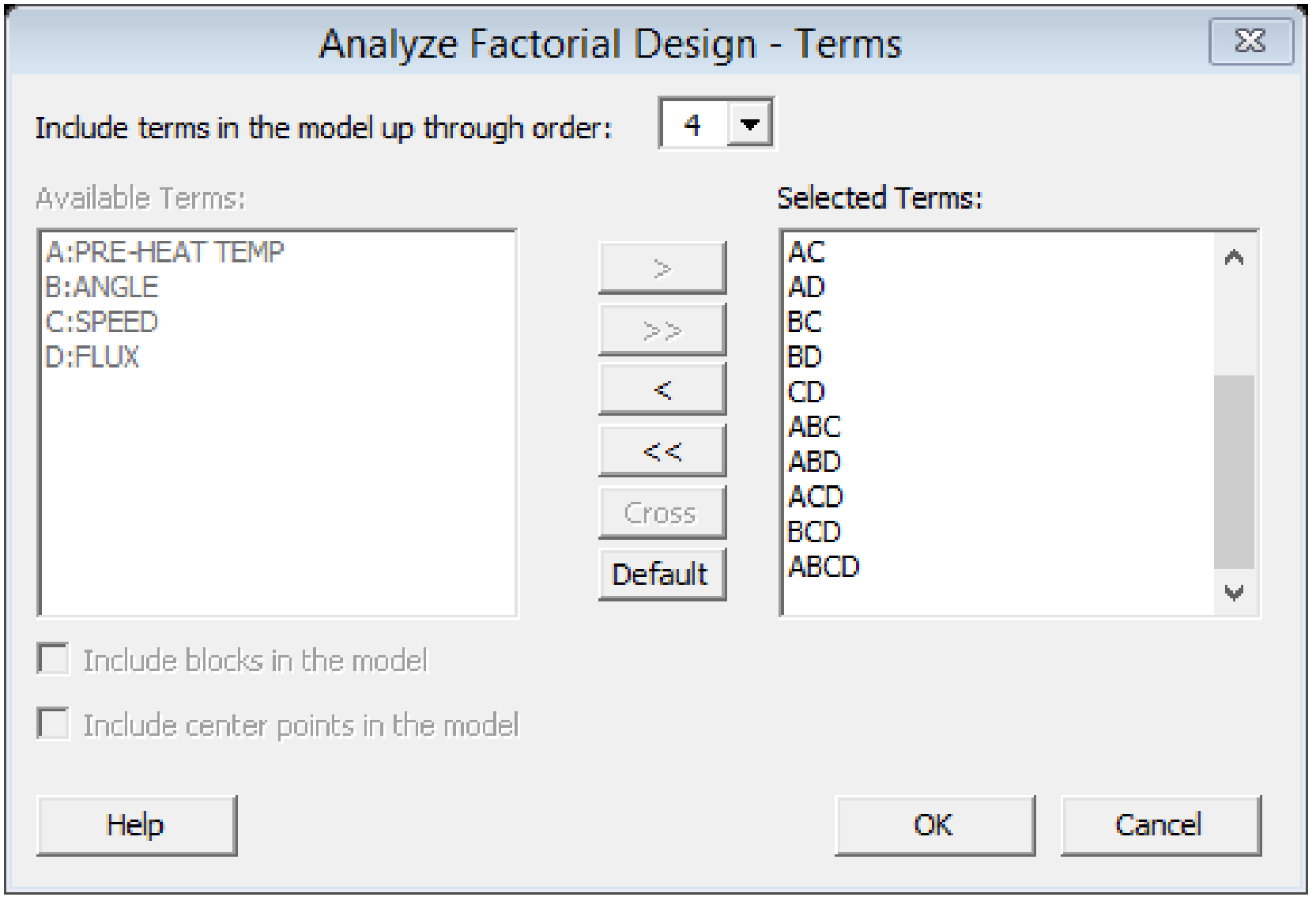
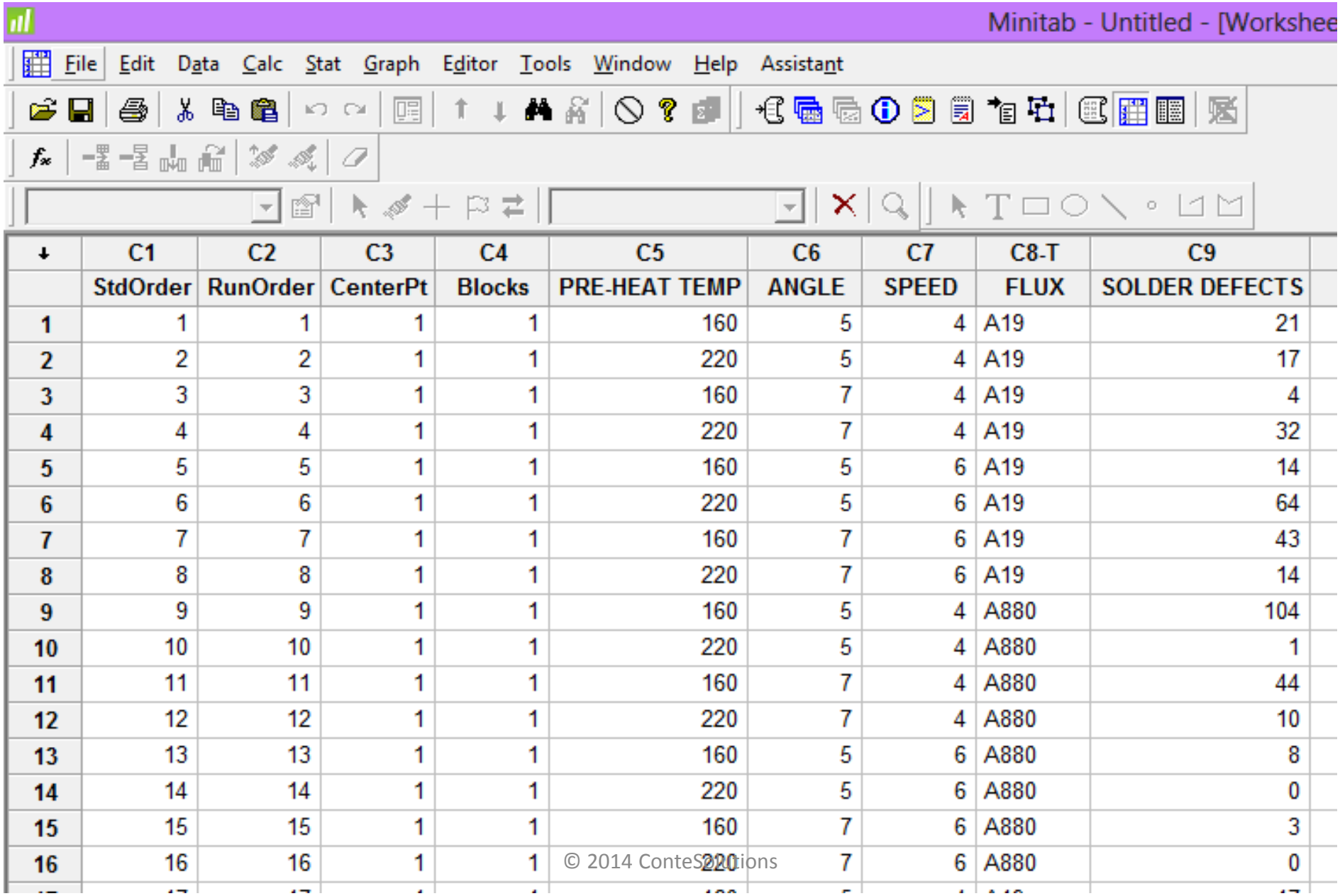
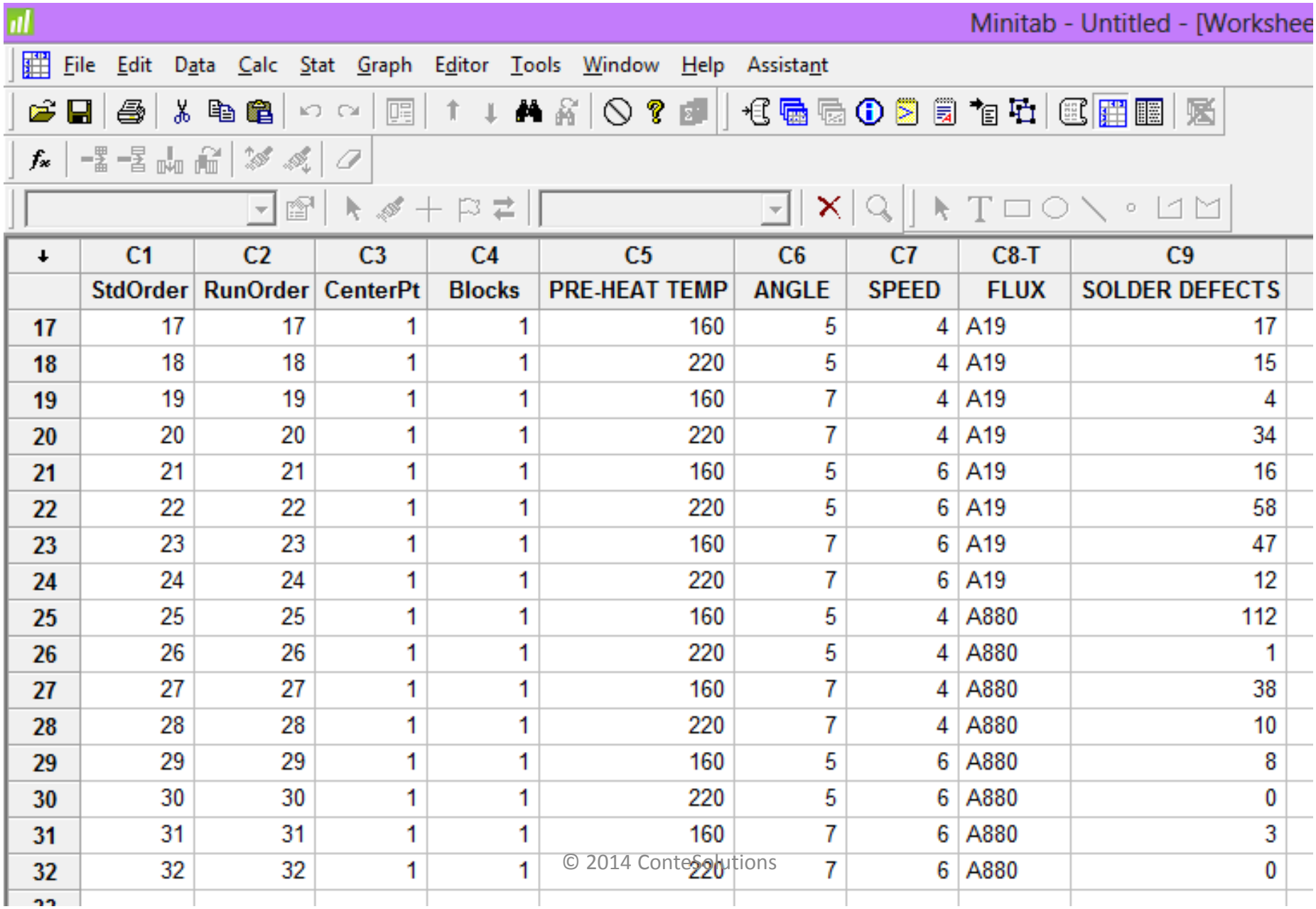




↓	C1	C2	C3	C4	C5	C6	C7	C8-T	C9
	StdOrder	RunOrder	CenterPt	Blocks	PRE-HEAT TEMP	ANGLE	SPEED	FLUX	SOLDER DEFECTS
1	1	1	1	1	160	5	4	A19	21
2	2	2	1	1	220	5	4	A19	17
3	3	3	1	1	160	7	4	A19	4
4	4	4	1	1	220	7	4	A19	32
5	5	5	1	1	160	5	6	A19	14
6	6	6	1	1	220	5	6	A19	64
7	7	7	1	1	160	7	6	A19	43
8	8	8	1	1	220	7	6	A19	14
9	9	9	1	1	160	5	4	A880	104
10	10	10	1	1	220	5	4	A880	1
11	11	11	1	1	160	7	4	A880	44
12	12	12	1	1	220	7	4	A880	10
13	13	13	1	1	160	5	6	A880	8
14	14	14	1	1	220	5	6	A880	0
15	15	15	1	1	160	7	6	A880	3
16	16	16	1	1	220	7	6	A880	0



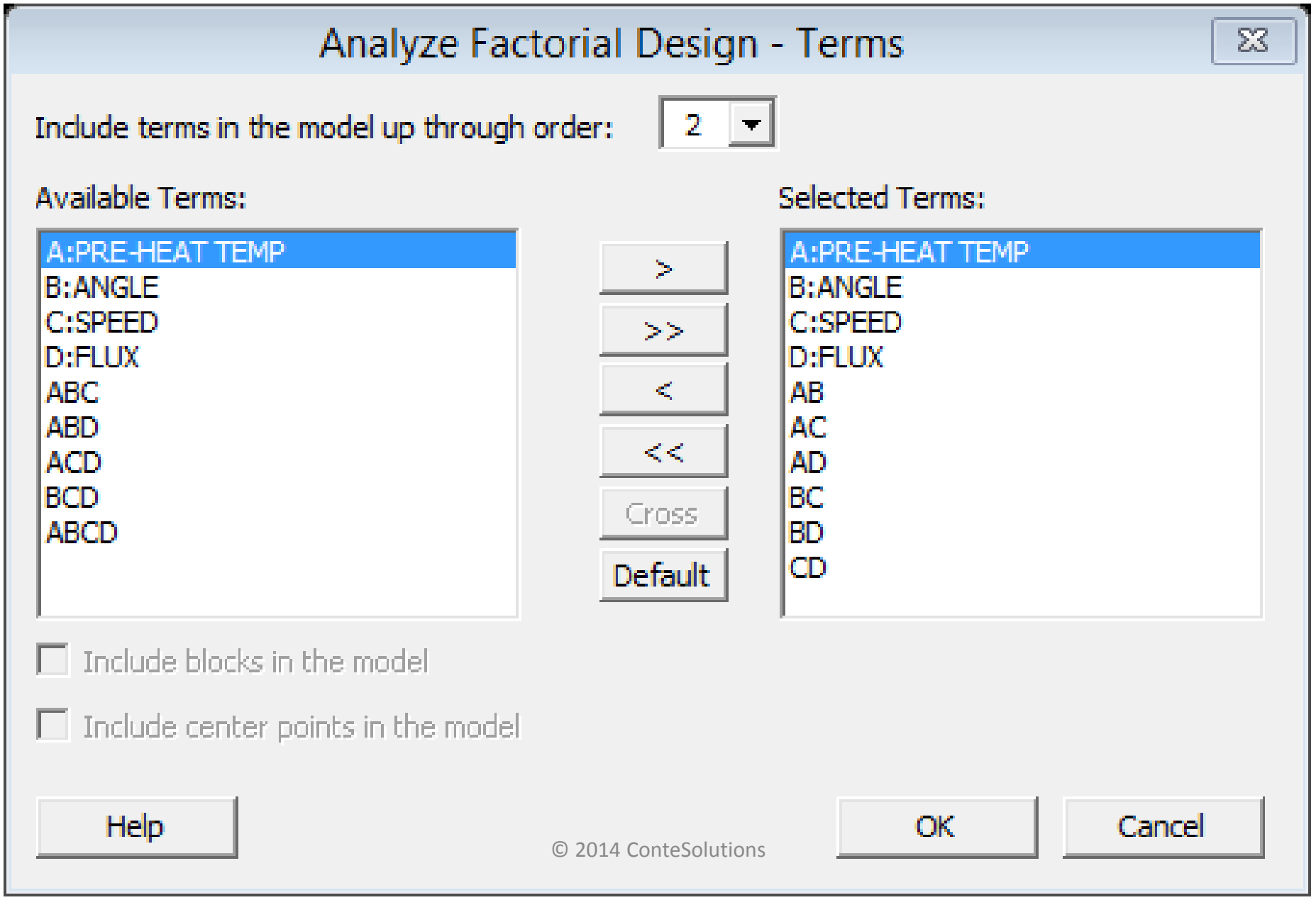
↓	C1	C2	C3	C4	C5	C6	C7	C8-T	C9
	StdOrder	RunOrder	CenterPt	Blocks	PRE-HEAT TEMP	ANGLE	SPEED	FLUX	SOLDER DEFECTS
17	17	17	1	1	160	5	4	A19	17
18	18	18	1	1	220	5	4	A19	15
19	19	19	1	1	160	7	4	A19	4
20	20	20	1	1	220	7	4	A19	34
21	21	21	1	1	160	5	6	A19	16
22	22	22	1	1	220	5	6	A19	58
23	23	23	1	1	160	7	6	A19	47
24	24	24	1	1	220	7	6	A19	12
25	25	25	1	1	160	5	4	A880	112
26	26	26	1	1	220	5	4	A880	1
27	27	27	1	1	160	7	4	A880	38
28	28	28	1	1	220	7	4	A880	10
29	29	29	1	1	160	5	6	A880	8
30	30	30	1	1	220	5	6	A880	0
31	31	31	1	1	160	7	6	A880	3
32	32	32	1	1	220	7	6	A880	0

ANOVA: SOLDER DEFECTS versus FLUX, SPEED, ANGLE, PRE-HEAT TEMP
**** ALL EFFECTS AND INTERACTION TERMS ANALYZED**

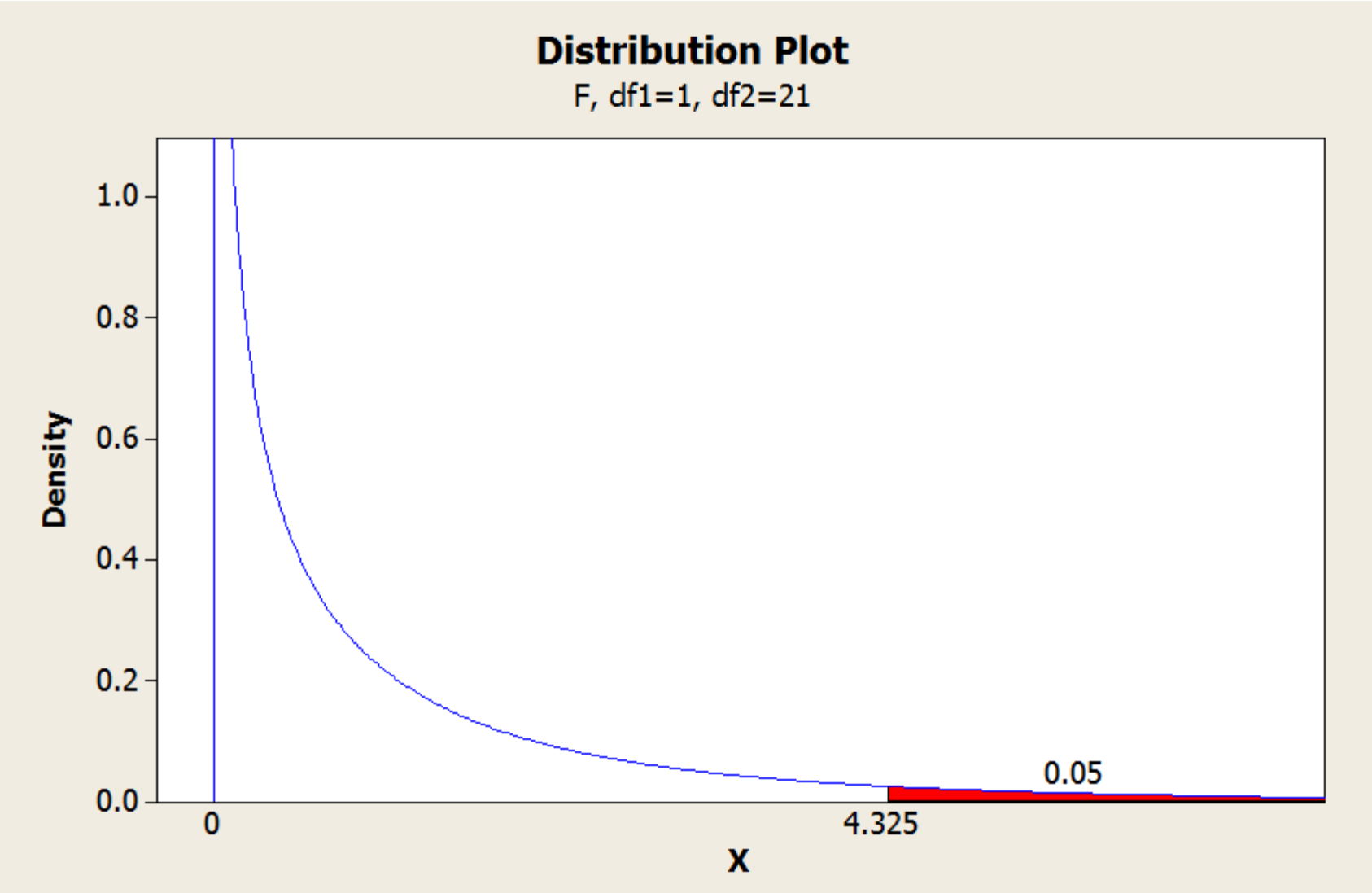
Factor	Type	Levels	Values
FLUX	fixed	2	A19, A880
SPEED	fixed	2	4, 6
ANGLE	fixed	2	5, 7
PRE-HEAT TEMP	fixed	2	160, 220

Analysis of Variance for SOLDER DEFECTS

Source	DF	SS	MS	F	P
FLUX	1	153.1	153.1	26.63	0.000
SPEED	1	946.1	946.1	164.54	0.000
ANGLE	1	780.1	780.1	135.67	0.000
PRE-HEAT TEMP	1	1485.1	1485.1	258.28	0.000
FLUX*SPEED	1	5565.1	5565.1	967.85	0.000
FLUX*ANGLE	1	276.1	276.1	48.02	0.000
FLUX*PRE-HEAT TEMP	1	4465.1	4465.1	776.54	0.000
SPEED*ANGLE	1	136.1	136.1	23.67	0.000
SPEED*PRE-HEAT TEMP	1	1653.1	1653.1	287.50	0.000
ANGLE*PRE-HEAT TEMP	1	153.1	153.1	26.63	0.000
FLUX*SPEED*ANGLE	1	666.1	666.1	115.85	0.000
FLUX*SPEED*PRE-HEAT TEMP	1	2415.1	2415.1	420.02	0.000
FLUX*ANGLE*PRE-HEAT TEMP	1	2016.1	2016.1	350.63	0.000
SPEED*ANGLE*PRE-HEAT TEMP	1	4095.1	4095.1	712.20	0.000
FLUX*SPEED*ANGLE*PRE-HEAT TEMP	1	190.1	190.1	33.07	0.000



SOURCE	SS	df	MS	F_0	F critical
Factor A	*153.125*	*1*	*153.125*	*<1*	*4.32 - Not Sig.*
Factor B	*946.125*	*1*	*946.125*	*2.10*	*4.32 - Not Sig.*
Factor C	*780.125*	*1*	*780.125*	*1.73*	*4.32 - Not Sig.*
Factor D	*1485.125*	*1*	*1485.125*	*3.29*	*4.32 - Not Sig.*
AB	***5565.125***	***1***	***5565.125***	***12.33***	***4.32 - Significant***
AC	*276.125*	*1*	*276.125*	*<1*	*4.32 - Not Sig.*
AD	***4465.125***	***1***	***4465.125***	***9.90***	***4.32 - Significant***
BC	*136.125*	*1*	*136.125*	*<1*	*4.32 - Not Sig.*
BD	*1653.125*	*1*	*1653.125*	*3.66*	*4.32 - Not Sig.*
CD	*153.125*	*1*	*153.125*	*<1*	*4.32 - Not Sig.*
Error	*9474.625*	*21*	*451.173*	-	
Total	*25087.875*	*31*	-	-	

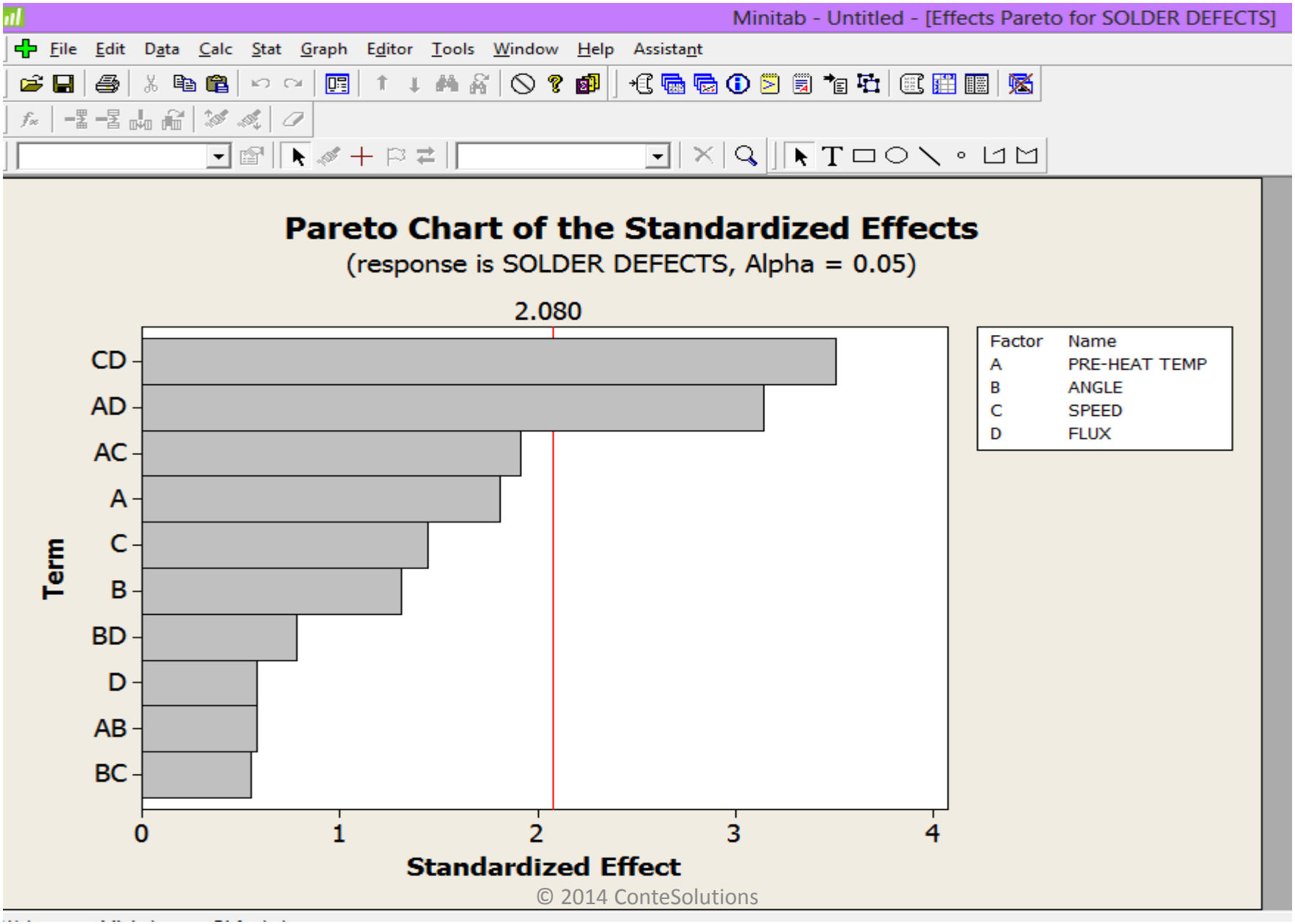


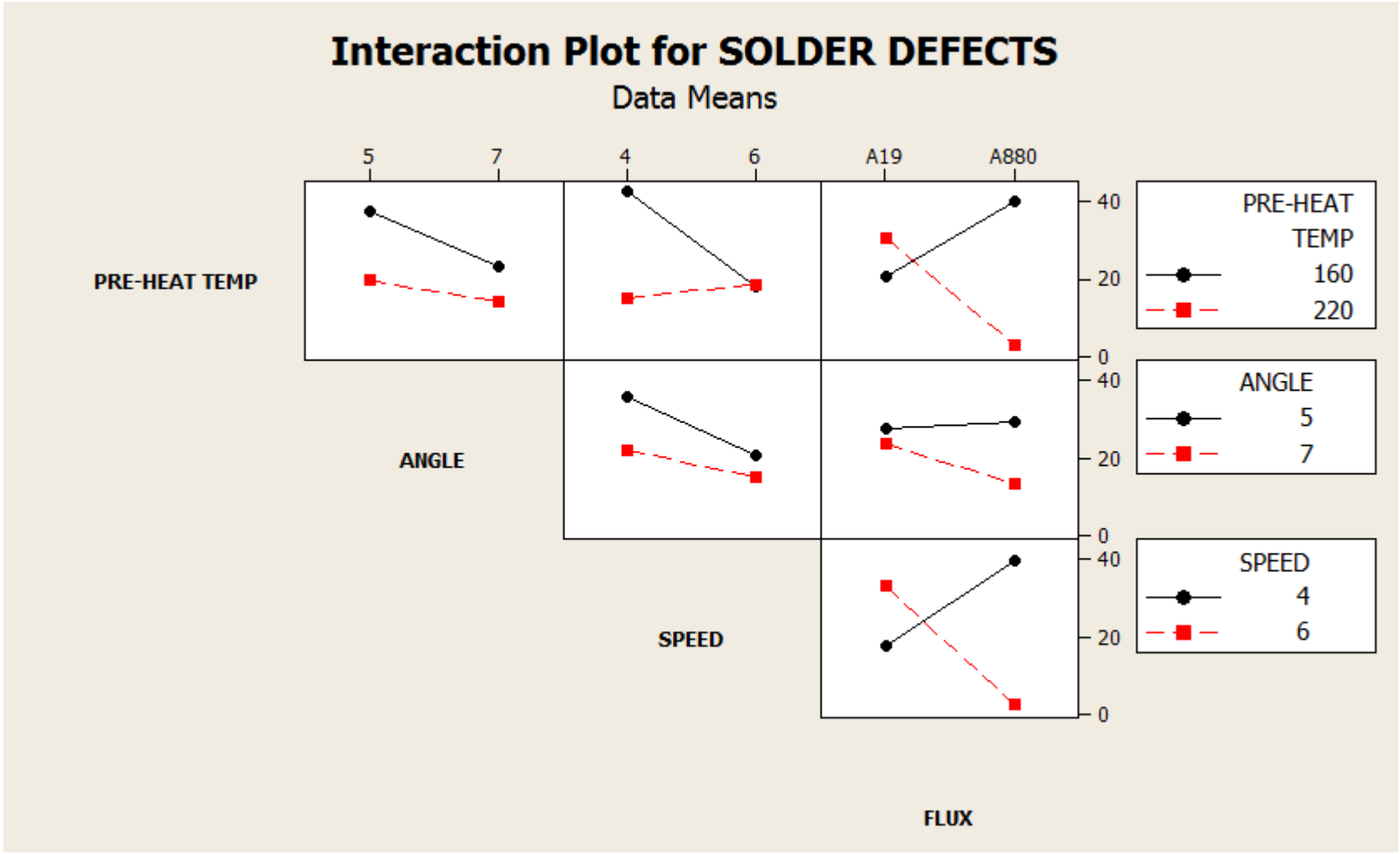
Analysis of Variance for SOLDER DEFECTS (coded units)

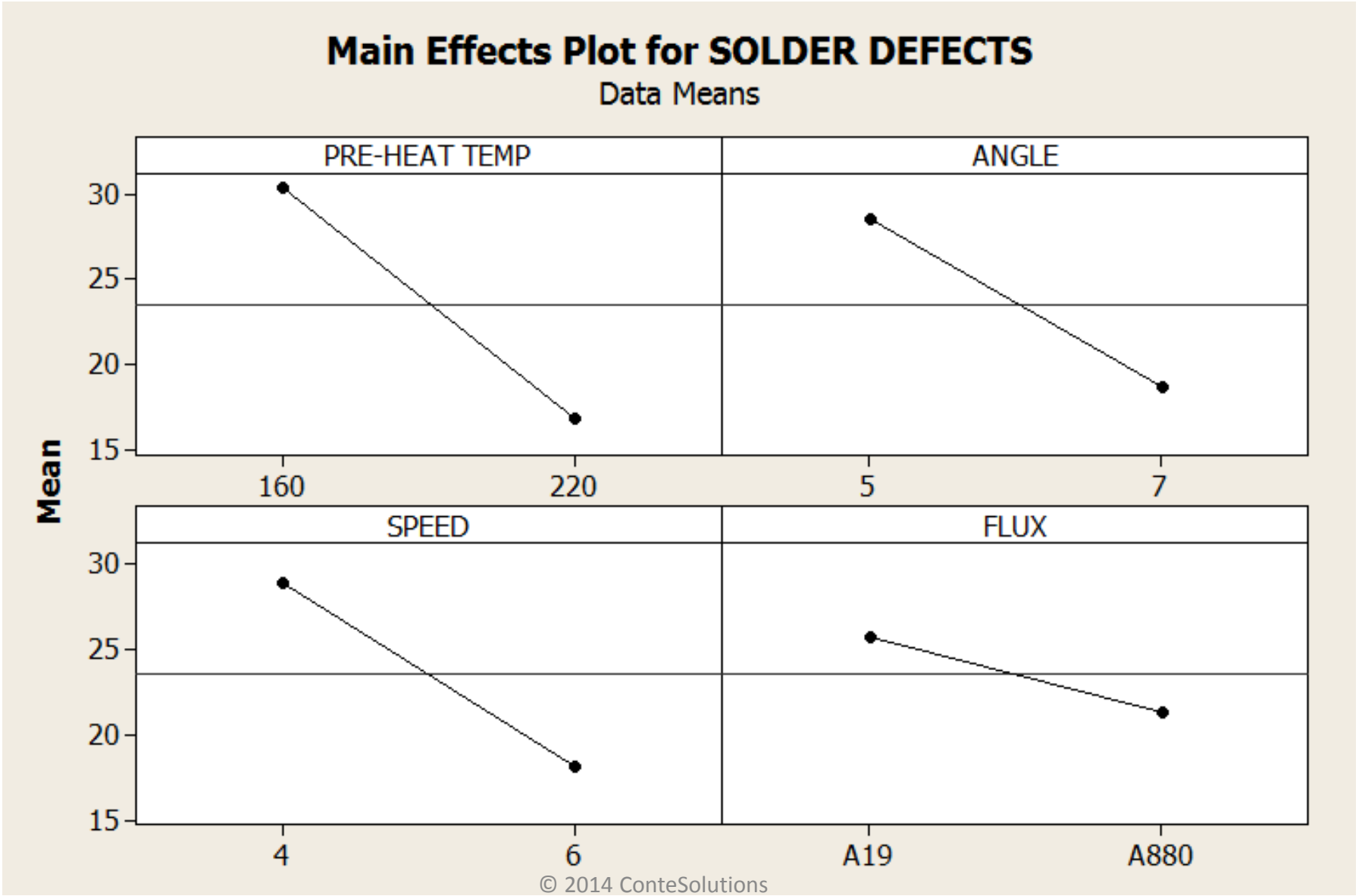
Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	4	3364.5	3364.5	841.13	1.86	0.154
PRE-HEAT TEMP	1	1485.1	1485.1	1485.12	3.29	0.084
ANGLE	1	780.1	780.1	780.12	1.73	0.203
SPEED	1	946.1	946.1	946.13	2.10	0.162
FLUX	1	153.1	153.1	153.12	0.34	0.566
2-Way Interactions	6	12248.8	12248.8	2041.46	4.52	0.004
PRE-HEAT TEMP*ANGLE	1	153.1	153.1	153.12	0.34	0.566
PRE-HEAT TEMP*SPEED	1	1653.1	1653.1	1653.12	3.66	0.069
PRE-HEAT TEMP*FLUX	1	4465.1	4465.1	4465.12	9.90	0.005
ANGLE*SPEED	1	136.1	136.1	136.12	0.30	0.589
ANGLE*FLUX	1	276.1	276.1	276.13	0.61	0.443
SPEED*FLUX	1	5565.1	5565.1	5565.13	12.33	0.002
Residual Error	21	9474.6	9474.6	451.17		
Lack of Fit	5	9382.6	9382.6	1876.53	326.35	0.000
Pure Error	16	92.0	92.0	5.75		
Total	31	25087.9				

Estimated Coefficients for SOLDER DEFECTS using data in uncoded units

Term	Coef
Constant	496.125
PRE-HEAT TEMP	-1.86250
ANGLE	-29.1042
SPEED	-63.3333
FLUX	156.187
PRE-HEAT TEMP*ANGLE	0.072917
PRE-HEAT TEMP*SPEED	0.239583
PRE-HEAT TEMP*FLUX	-0.393750
ANGLE*SPEED	2.06250
ANGLE*FLUX	-2.93750
SPEED*FLUX	-13.1875







Given all of this information, the best levels of the three significant factors Flux (A), Belt Speed (B), and Pre-Heat Temperature (D) (when conducting the ANOVA through the 2nd order interaction only) are:

A880 Flux, 6 ft./min. Belt Speed,
and 220° F Pre-heat Temperature

Since Angle of Incline (C) was not found to be significant with regard to solder defects, it cannot be set based on solder defects and should instead be set based on some other predictable result.