

Central Limit Theorem

- First introduced on page 27 and 28 of Tab 6, but applicable when confidence interval introduced on page 87 of Tab 6
- 1965 text by Duncan entitled Quality Control and Industrial Statistics, p 116
- Theorem 6.3 – The distribution of sample means approaches the form of a normal distribution as the size of the sample increases
- Theorem 6.7 – The distribution of sample variances is distributed as a chi-square distribution

Central Limit Theorem

- 1965 text by Duncan entitled Quality Control and Industrial Statistics, p 116
- THEOREM 6.3 – The form of the distribution of **sample means** approaches the form of a normal distribution as the size of the sample increases
- Difference data and statistics
- Distribution of sample means
- Distribution of sample variances
- SPC implications

Central Limit Theorem

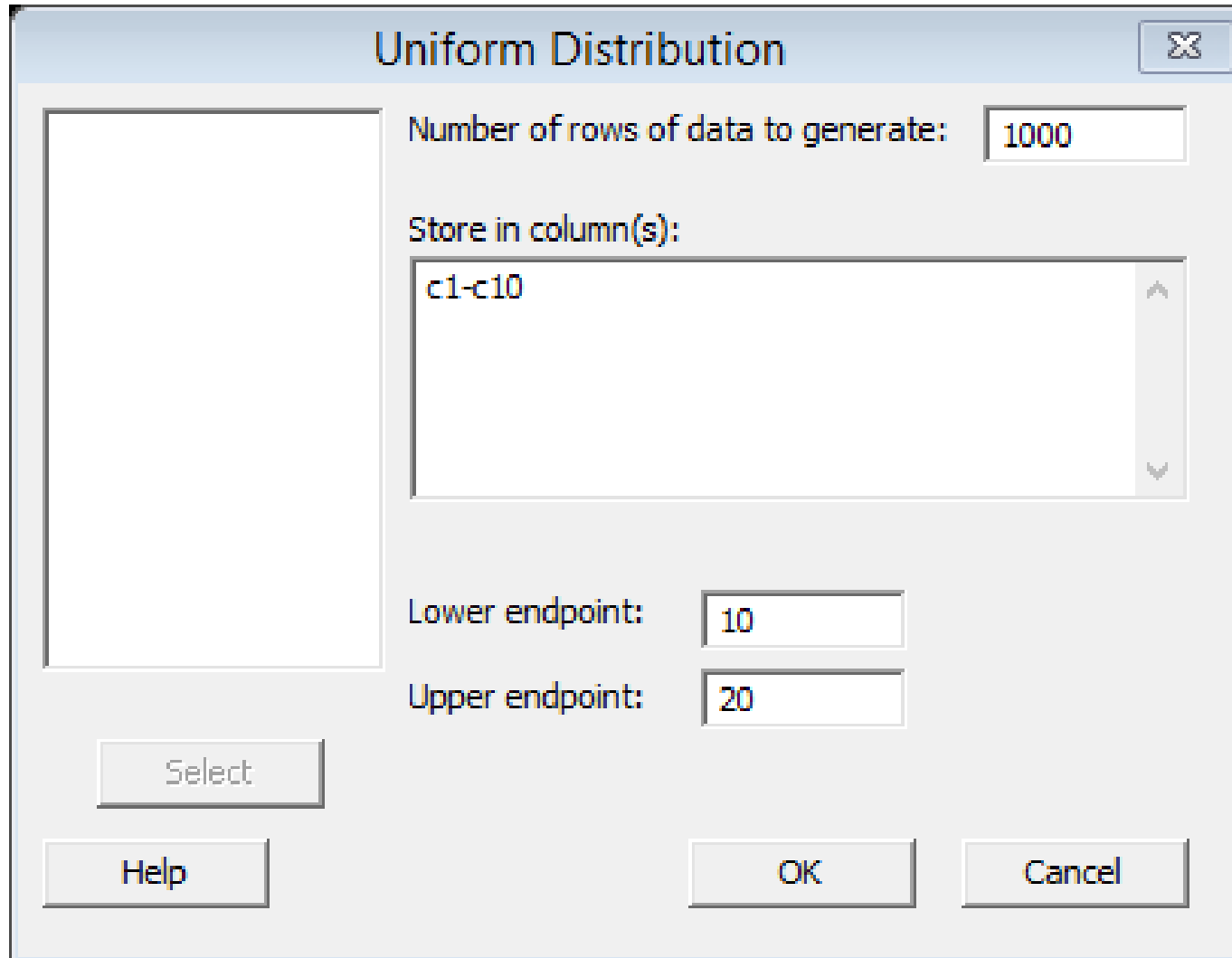
- Three important properties stem from the Central Limit Theorem on sample means:
 - (1) The distribution of all possible sample means will have an **mean** equal to the average of the population from which the samples were selected.
 - (2) The distribution of all possible sample means will have a **standard deviation** equal to the standard deviation of the population from which the sample averages are selected, divided by the square root of the sample size.
 - (3) The **distribution** of all possible sample means will approach a normal distribution as the size of n increases

Central Limit Theorem Demo

- Minitab – create data in col 1-10 uniform 10 to 20, 1000 data points each
- Stack in c15 / histogram, mean, std dev
- Calculate sample means in c12
- Histogram, note shape, mean, and std deviation
- SPC implications

Mintab

Calc/Random data/Uniform



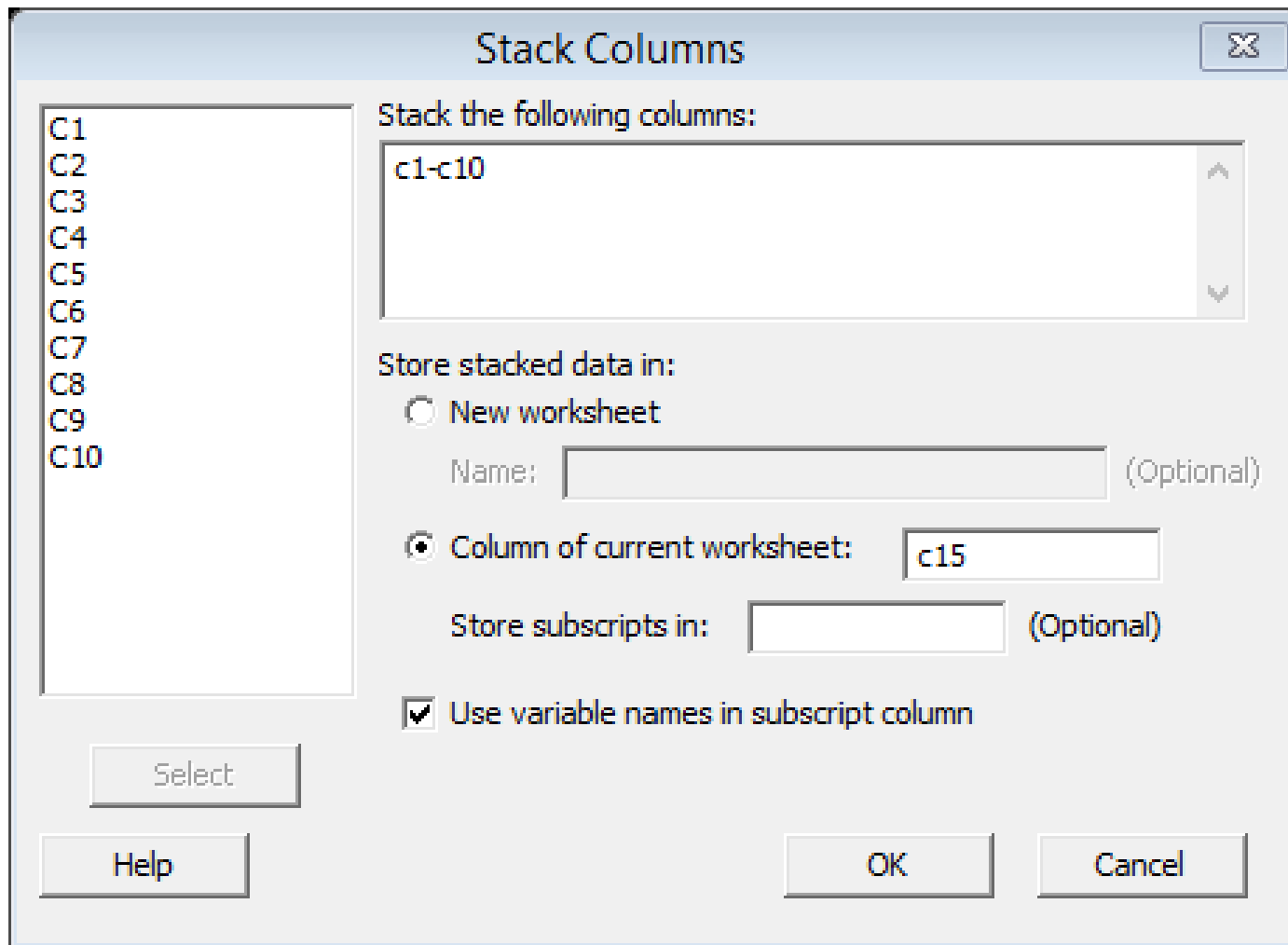
Minitab - Untitled - [Worksheet 1 ***]

File Edit Data Calc Stat Graph Editor Tools Window Help Assistant

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	C17	C18	C19	C20
1	13.7394	11.4344	13.0385	10.5340	14.5848	15.5460	11.1235	11.3220	16.9295	18.1625										
2	16.2319	15.4955	12.3078	18.8850	16.9217	14.3607	17.9195	13.3349	16.1953	17.7728										
3	18.0617	11.0845	11.0539	13.7472	15.8937	10.7870	17.0857	17.8842	13.4744	12.0224										
4	10.9107	18.5950	15.2812	10.3060	19.4350	10.0821	14.9065	13.7398	13.5805	12.2169										
5	13.6056	18.0252	15.8651	16.0796	13.6958	11.4732	10.0948	14.9676	16.5422	10.6756										
6	10.9809	18.5080	16.6057	18.4397	14.4278	17.8053	17.5694	18.9856	19.2336	12.3720										
7	15.0792	17.0018	14.9954	12.3930	10.8858	10.2818	12.9156	16.1446	11.4184	18.9465										
8	16.6454	13.9537	14.2747	19.6264	11.0427	11.6151	18.4247	19.5422	11.0596	19.5349										
9	14.2653	15.9016	13.1894	19.0795	19.6148	17.1200	18.7501	16.3233	10.7312	16.5679										
10	14.3646	14.3800	10.2216	15.9380	15.3321	11.1590	14.5737	17.5766	10.6144	17.4811										
11	11.7517	14.4493	12.6056	10.3028	12.0689	19.9813	19.2188	18.8404	19.1026	12.7763										
12	15.3763	12.9783	13.7622	13.6402	15.7265	10.5782	15.9943	19.4337	18.1619	17.3111										
13	19.9058	19.4693	18.1437	19.0297	10.0581	10.8909	15.9175	16.7157	19.8763	10.6102										
14	17.2923	12.3213	10.7174	10.1163	11.2435	16.3486	18.1922	11.0612	13.6767	17.1317										
15	15.7845	10.3598	19.7207	13.0637	16.5197	17.0679	15.1457	12.3258	17.8355	11.9931										
16	10.7413	19.9654	12.7260	16.7650	17.4585	11.8476	16.5957	14.1240	13.0097	13.0862										
17	16.7888	13.6791	10.0678	13.2966	19.9527	15.9104	17.9101	13.5263	13.4626	17.2842										
18	19.5271	19.4389	12.0135	16.2341	15.8074	19.4996	17.4174	15.5827	11.1906	17.1462										
19	17.9880	15.8639	15.3060	19.8212	16.9899	14.1753	17.9816	12.4758	13.8641	11.2309										
20	18.5855	12.6658	13.0657	19.9505	18.8841	18.2360	13.9829	13.4742	19.0098	18.3792										
21	12.5187	12.4253	19.7760	10.4514	12.4960	19.7166	19.1135	12.7520	16.1434	16.4993										
22	12.1371	16.2856	16.3093	10.4556	17.4403	13.7802	17.0672	17.3799	13.8493	11.5139										
23	17.0651	13.3307	19.4313	16.1429	18.7099	15.2057	13.4731	11.6780	16.9884	16.8375										
24	15.7050	17.1380	13.0646	11.7585	18.7630	13.6540	18.2441	11.3774	16.9907	14.2138										
25	10.0000	17.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000										

Current Worksheet: Worksheet 1

Data/Stack



The image shows a 'Stack Columns' dialog box with a list of columns on the left and configuration options on the right. The list includes C1 through C10. The 'Stack the following columns:' field contains 'c1-c10'. Under 'Store stacked data in:', the 'Column of current worksheet:' option is selected with 'c15' entered. The 'Use variable names in subscript column' checkbox is checked. Buttons for 'Select', 'Help', 'OK', and 'Cancel' are present.

Stack Columns [Close]

Stack the following columns:

c1-c10

Store stacked data in:

New worksheet

Name: (Optional)

Column of current worksheet:

Store subscripts in: (Optional)

Use variable names in subscript column

Select

Help

OK Cancel

Minitab - Untitled - [Worksheet 1 ***]

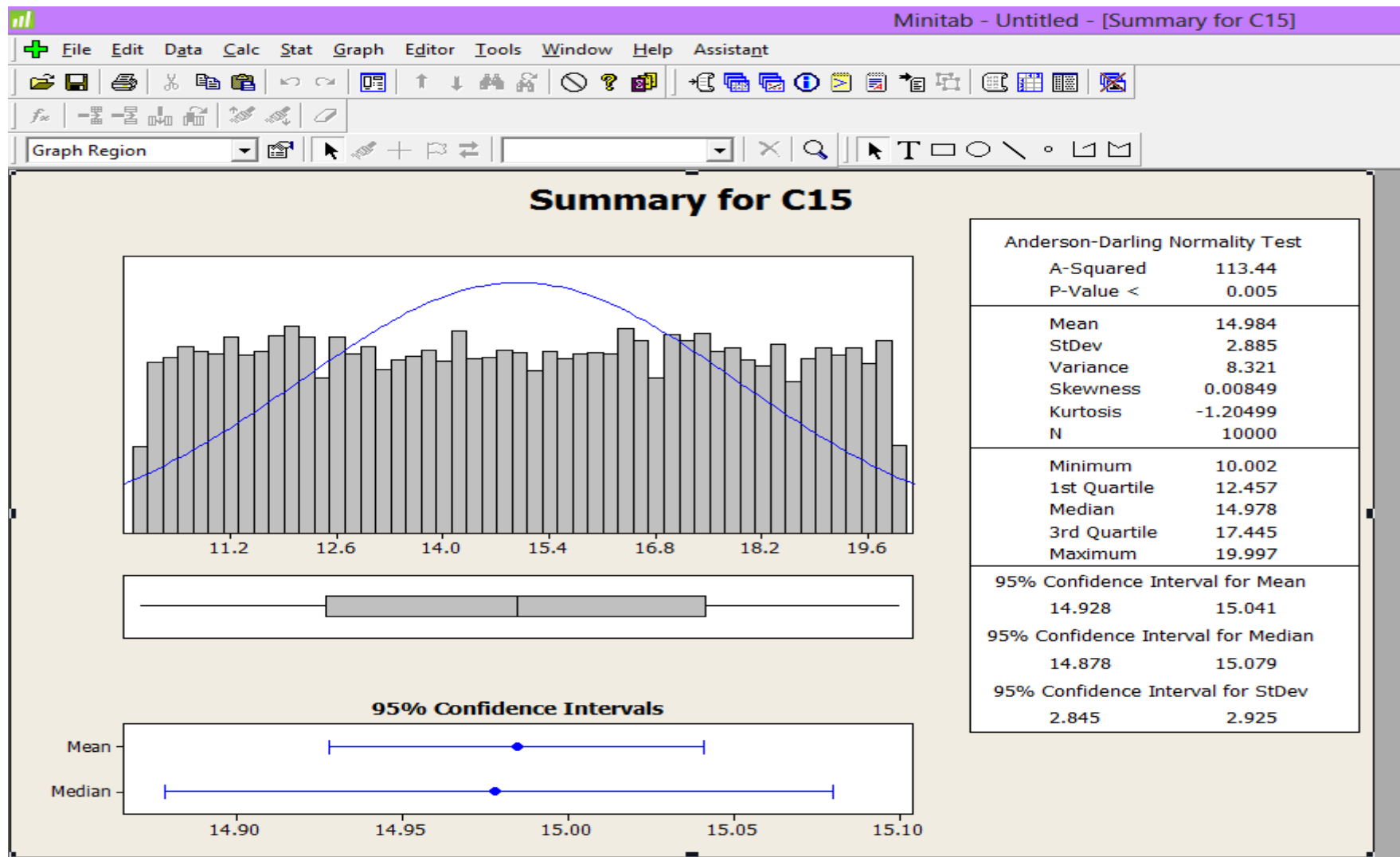
File Edit Data Calc Stat Graph Editor Tools Window Help Assistant

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	C17	C18	C19	C20
1	13.7394	11.4344	13.0385	10.5340	14.5848	15.5460	11.1235	11.3220	16.9295	18.1625					13.7394					
2	16.2319	15.4955	12.3078	18.8850	16.9217	14.3607	17.9195	13.3349	16.1953	17.7728					16.2319					
3	18.0617	11.0845	11.0539	13.7472	15.8937	10.7870	17.0857	17.8842	13.4744	12.0224					18.0617					
4	10.9107	18.5950	15.2812	10.3060	19.4350	10.0821	14.9065	13.7398	13.5805	12.2169					10.9107					
5	13.6056	18.0252	15.8651	16.0796	13.6958	11.4732	10.0948	14.9676	16.5422	10.6756					13.6056					
6	10.9809	18.5080	16.6057	18.4397	14.4278	17.8053	17.5694	18.9856	19.2336	12.3720					10.9809					
7	15.0792	17.0018	14.9954	12.3930	10.8858	10.2818	12.9156	16.1446	11.4184	18.9465					15.0792					
8	16.6454	13.9537	14.2747	19.6264	11.0427	11.6151	18.4247	19.5422	11.0596	19.5349					16.6454					
9	14.2653	15.9016	13.1894	19.0795	19.6148	17.1200	18.7501	16.3233	10.7312	16.5679					14.2653					
10	14.3646	14.3800	10.2216	15.9380	15.3321	11.1590	14.5737	17.5766	10.6144	17.4811					14.3646					
11	11.7517	14.4493	12.6056	10.3028	12.0689	19.9813	19.2188	18.8404	19.1026	12.7763					11.7517					
12	15.3763	12.9783	13.7622	13.6402	15.7265	10.5782	15.9943	19.4337	18.1619	17.3111					15.3763					
13	19.9058	19.4693	18.1437	19.0297	10.0581	10.8909	15.9175	16.7157	19.8763	10.6102					19.9058					
14	17.2923	12.3213	10.7174	10.1163	11.2435	16.3486	18.1922	11.0612	13.6767	17.1317					17.2923					
15	15.7845	10.3598	19.7207	13.0637	16.5197	17.0679	15.1457	12.3258	17.8355	11.9931					15.7845					
16	10.7413	19.9654	12.7260	16.7650	17.4585	11.8476	16.5957	14.1240	13.0097	13.0862					10.7413					
17	16.7888	13.6791	10.0678	13.2966	19.9527	15.9104	17.9101	13.5263	13.4626	17.2842					16.7888					
18	19.5271	19.4389	12.0135	16.2341	15.8074	19.4996	17.4174	15.5827	11.1906	17.1462					19.5271					
19	17.9880	15.8639	15.3060	19.8212	16.9899	14.1753	17.9816	12.4758	13.8641	11.2309					17.9880					
20	18.5855	12.6658	13.0657	19.9505	18.8841	18.2360	13.9829	13.4742	19.0098	18.3792					18.5855					
21	12.5187	12.4253	19.7760	10.4514	12.4960	19.7166	19.1135	12.7520	16.1434	16.4993					12.5187					
22	12.1371	16.2856	16.3093	10.4556	17.4403	13.7802	17.0672	17.3799	13.8493	11.5139					12.1371					
23	17.0651	13.3307	19.4313	16.1429	18.7099	15.2057	13.4731	11.6780	16.9884	16.8375					17.0651					
24	15.7050	17.1380	13.0646	11.7585	18.7630	13.6540	18.2441	11.3774	16.9907	14.2138					15.7050					
25	10.0043	17.3054	10.0070	10.0047	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000					10.0043					

Create new worksheet from current worksheet data based on a condition

Stat/Basic Statistics/Graphical Summary/C15

Note: Mean = 14.984 and std dev = 2.885



Calc/Row Statistics

Row Statistics [Close]

C1
C2
C3
C4
C5
C6
C7
C8
C9
C10
C15

Statistic

Sum Median
 Mean Sum of squares
 Standard deviation N total
 Minimum N nonmissing
 Maximum N missing
 Range

Input variables:
c1-c10

Store result in: c12

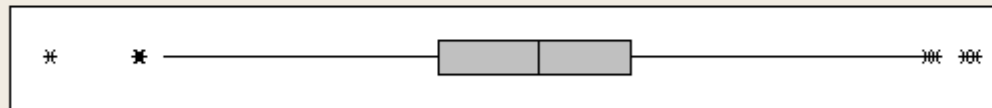
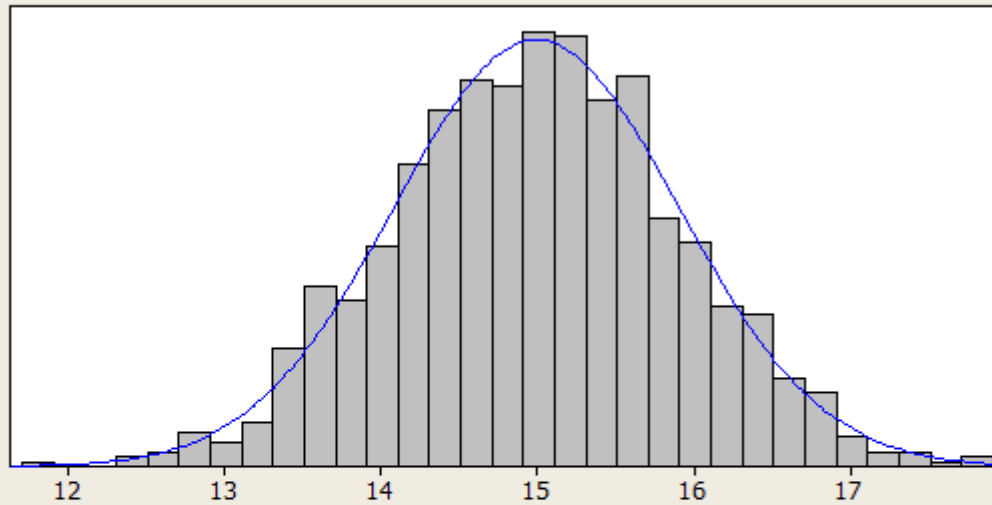
Select Help OK Cancel

Worksheet 1 ***												
↓	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
1	13.7394	11.4344	13.0385	10.5340	14.5848	15.5460	11.1235	11.3220	16.9295	18.1625		13.6415
2	16.2319	15.4955	12.3078	18.8850	16.9217	14.3607	17.9195	13.3349	16.1953	17.7728		15.9425
3	18.0617	11.0845	11.0539	13.7472	15.8937	10.7870	17.0857	17.8842	13.4744	12.0224		14.1095
4	10.9107	18.5950	15.2812	10.3060	19.4350	10.0821	14.9065	13.7398	13.5805	12.2169		13.9054
5	13.6056	18.0252	15.8651	16.0796	13.6958	11.4732	10.0948	14.9676	16.5422	10.6756		14.1025
6	10.9809	18.5080	16.6057	18.4397	14.4278	17.8053	17.5694	18.9856	19.2336	12.3720		16.4928
7	15.0792	17.0018	14.9954	12.3930	10.8858	10.2818	12.9156	16.1446	11.4184	18.9465		14.0062
8	16.6454	13.9537	14.2747	19.6264	11.0427	11.6151	18.4247	19.5422	11.0596	19.5349		15.5719
9	14.2653	15.9016	13.1894	19.0795	19.6148	17.1200	18.7501	16.3233	10.7312	16.5679		16.1543
10	14.3646	14.3800	10.2216	15.9380	15.3321	11.1590	14.5737	17.5766	10.6144	17.4811		14.1641
11	11.7517	14.4493	12.6056	10.3028	12.0689	19.9813	19.2188	18.8404	19.1026	12.7763		15.1098
12	15.3763	12.9783	13.7622	13.6402	15.7265	10.5782	15.9943	19.4337	18.1619	17.3111		15.2963
13	19.9058	19.4693	18.1437	19.0297	10.0581	10.8909	15.9175	16.7157	19.8763	10.6102		16.0617
14	17.2923	12.3213	10.7174	10.1163	11.2435	16.3486	18.1922	11.0612	13.6767	17.1317		13.8101
15	15.7845	10.3598	19.7207	13.0637	16.5197	17.0679	15.1457	12.3258	17.8355	11.9931		14.9816
16	10.7413	19.9654	12.7260	16.7650	17.4585	11.8476	16.5957	14.1240	13.0097	13.0862		14.6319
17	16.7888	13.6791	10.0678	13.2966	19.9527	15.9104	17.9101	13.5263	13.4626	17.2842		15.1879
18	19.5271	19.4389	12.0135	16.2341	15.8074	19.4996	17.4174	15.5827	11.1906	17.1462		16.3858
19	17.9880	15.8639	15.3060	19.8212	16.9899	14.1753	17.9816	12.4758	13.8641	11.2309		15.5697
20	18.5855	12.6658	13.0657	19.9505	18.8841	18.2360	13.9829	13.4742	19.0098	18.3792		16.6234
21	12.5187	12.4253	19.7760	10.4514	12.4960	19.7166	19.1135	12.7520	16.1434	16.4993		15.1892
22	12.1371	16.2856	16.3093	10.4556	17.4403	13.7802	17.0672	17.3799	13.8493	11.5139		14.6218
23	17.0654	12.0007	10.1010	10.1100	10.7000	15.0057	10.1704	14.0700	10.0004	10.0000		15.0000

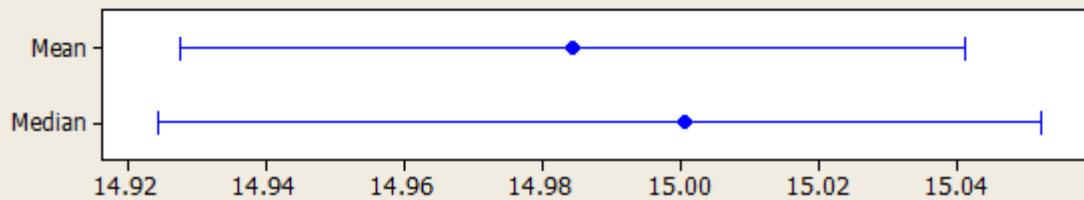
Stat/Basic Statistics/Graphical Summary/C15

Note: Mean = 14.984 and std dev = 0.913

Summary for C12



95% Confidence Intervals



Anderson-Darling Normality Test

A-Squared	0.14
P-Value	0.972

Mean	14.984
StDev	0.913
Variance	0.833
Skewness	-0.0077796
Kurtosis	-0.0918649
N	1000

Minimum	11.881
1st Quartile	14.356
Median	15.000
3rd Quartile	15.593
Maximum	17.786

95% Confidence Interval for Mean

14.928	15.041
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95% Confidence Interval for Median

14.924	15.052
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95% Confidence Interval for StDev

0.875	0.955
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(1) The distribution of all possible sample means will have an mean equal to the average of the population from which the samples were selected.

- Mean of uniform data = 14.984
- Mean of sample means = 14.984

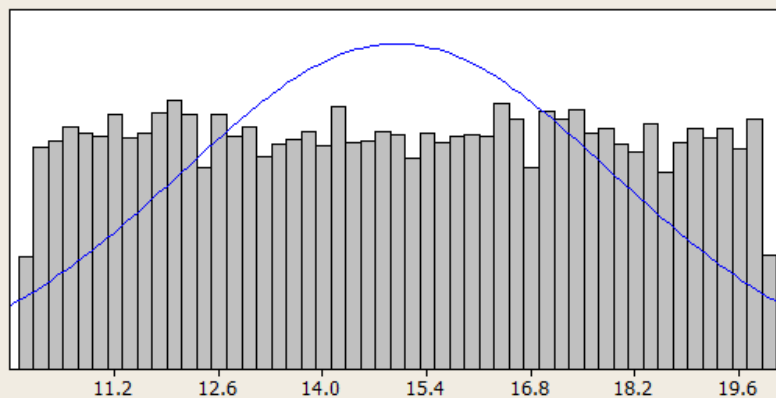
(2) The distribution of all possible sample means will have a standard deviation equal to the standard deviation of the population from which the sample averages are selected, divided by the square root of the sample size.

- Standard deviation of uniform data = 2.885
- Standard Deviation of sample means = 0.913

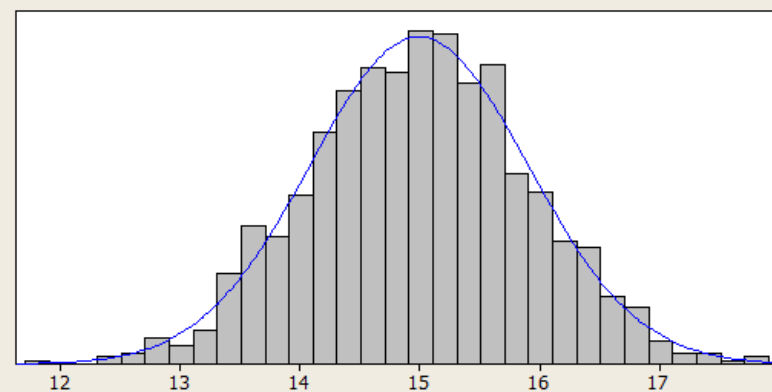
$$\sigma_{\bar{X}} = \frac{\sigma_X}{\sqrt{n}} = \frac{2.885}{\sqrt{10}} = 0.91$$

(3) The distribution of all possible sample means will approach a normal distribution as the size of n increases

Summary for C15



Summary for C12



CLT Implication for X-bar and R chart

- If the data (x) from your process historically has a mean of 10 and a standard deviation of 1.0 and your sample size is 5
- The 3-sigma control limits on the x-bar statistic will be:

$$\bar{X} \pm 3\sigma_{\bar{X}} = \bar{X} \pm 3 \frac{\sigma_X}{\sqrt{n}} = 10 \pm 3 \frac{1.0}{\sqrt{5}} = 10 \pm 1.34$$

- 11.34 and 8.66