Introduction
To Quality Trend
Charts

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Quality Assurance for Bellcore Clients

Bell Communications Research (Bellcore) Quality Assurance (QA) is responsible for checking and reporting on the status of the quality of certain products and services purchased by Bellcore clients. The status of quality is determined from the customer's viewpoint and published in quality reports. The publication of these reports is one service provided by Bellcore-QA to Bellcore clients. These reports help customers make decisions about suppliers and/or their products based on quality. Quality reports are also used to provide customer feedback to the supplier on the quality of the products or services in the reports.

Quality surveillance inspection of new products or services by Bellcore-QA at supplier locations consists of five phases: sampling, checking, defect assessment, quality rating, and reporting. This booklet provides an overview of the methods used in the quality rating and reporting phases.

The Quality Trend Chart (Trend Chart for short) shown in Illustration 1 is the graphical method used to illustrate quality rating and reporting. The Trend Chart is composed of a time series of "T" Plots connected by a trend line. The "T" Plot for a given period is an interval estimate of quality for that period. The trend line shows the trend in the long run average for quality. The next section describes in detail the meaning of the symbols used in the Trend Chart. The subsequent section discusses how to interpret Trend Charts, particularly as they exhibit movement over time. The last section provides examples of Trend Charts which exhibit various types of process quality behavior. It is hoped that these examples will provide assistance in reviewing and interpreting the Trend Charts.

ILLUSTRATION 1

![Quality Trend Chart](chart.png)
What are Quality Trend Charts?

The purpose of any quality rating plan is to evaluate product quality and document the results for information and decision making. Such plans generally use statistical methods to interpret information obtained from a sampling of a product. This plan uses an enhanced version of the Quality Measurement Plan (QMP). QMP utilizes not only data from the current rating period, but also makes an assessment of how well data from 5 previous periods are related to CURRENT QUALITY. Using that assessment, QMP then optimally combines current data with past data to make a better statistical evaluation of the CURRENT QUALITY of a product. How much better the evaluation is depends on the assessed value of the data in the 5 previous periods.

The Trend Charts provide the Bellcore clients with a series of "I" plots connected by a line that show current product quality as well as quality trends over time. Look at the Trend Chart in Illustration 1.

The Time Scale

The numbers running horizontally across the top of the Trend Chart represent rating periods. Quality in each rating period is shown by the "I" Plot below the period numbers.

The Index Scale

On the left side of the Trend Chart is the Index Scale for measuring quality. This scale runs from 0 to 5, with 1 representing Standard Quality. The index is a ratio of actual quality, in terms of some measure of defectiveness, to Standard Quality. When Measured Quality is equal to Standard Quality, the index is 1. Values of the index less than 1 represent better-than-standard quality; values greater than 1 show worse-than-standard quality. For example, the value 2 would mean that there are twice as many defects observed as expected under the quality standard.

The Standard Quality is specified below each Trend Chart in standard (or expected) defects per unit.

The Symbols Within The Trend Charts

For each period in the Trend Chart in Illustration 1 there is an "I" Plot consisting of four elements: a vertical line segment with two end points, a horizontal dash, an X, and a circle. The circles in the figures are connected from period to period by lines which run generally horizontally across the Trend Chart. Let's take a closer look at one of the "I" Plots as shown in Illustration 2 and see what each symbol denotes:

**ILLUSTRATION 2**

- **LONG RUN AVERAGE (the circle):** The data for 6 rating periods (5 prior periods plus the current period) are combined into a summary index called the LONG RUN AVERAGE. This number is a weighted average of indices for the 6 periods. The weighting accounts for period-to-period variation in the sample size.

- **CURRENT SAMPLE INDEX (the X):** The quality index for the sample taken in the current rating period is called the CURRENT SAMPLE INDEX and is simply the ratio of the number of defects actually found in the sample to the number of defects expected at Standard Quality.

- **BEST MEASURE OF CURRENT QUALITY (the horizontal dash):** The BEST MEASURE OF CURRENT QUALITY is a weighted average of the LONG RUN AVERAGE and the CURRENT SAMPLE INDEX. It is the "best" estimate of the true (but unknown) quality of current production. The weights used in the average depend on the relative magnitudes of two sources of variation:
  
a. Process Variation (PV)—a measure of the period-to-period variation in the product's true quality.
  
b. Sampling Variation (SV)—a measure of the deviations between the quality in the sample and the product's true quality in a rating period.

These two sources of variation account for the total variation in CURRENT QUALITY.

Note that the BEST MEASURE OF CURRENT QUALITY will always fall between the circle, representing the LONG RUN AVERAGE, and the X, representing the CURRENT SAMPLE INDEX, because the BEST MEASURE OF CURRENT QUALITY is a weighted average of the two. The distance of the X and the circle on the index scale from the BEST MEASURE OF CURRENT QUALITY is an indication of which of the two carried more weight in the calculation of the BEST MEASURE. The closer one carried more weight for that period.
In fact, the BEST MEASURE OF CURRENT QUALITY has the form

\[ W [\text{LONG RUN AVERAGE}] + (1-W) [\text{CURRENT SAMPLE INDEX}] \]

where the weight, \( W \), is determined by

\[ W = \frac{SV}{SV + PV} \]

The bigger the sampling variation is, relative to the process variation, the more weight is put on the LONG RUN AVERAGE. The observed \% shrinkage, or movement, of the BEST MEASURE OF CURRENT QUALITY toward the LONG RUN AVERAGE (and away from the CURRENT SAMPLE INDEX) is defined by

\[ \% \text{ shrinkage} = W \times 100\% \]

When the sample is small, producing a large sampling variation, the BEST MEASURE gets most of its information from the LONG RUN AVERAGE and shrinks towards the LONG RUN AVERAGE. On the Trend Chart the BEST MEASURE will appear close to the LONG RUN AVERAGE. When the sample gets larger, the BEST MEASURE gets more information from the sample and shrinks less toward the LONG RUN AVERAGE. As sampling variation gets small enough to be in the same numerical range as the process variation, \( W \) approaches \( \frac{1}{2} \) and the BEST MEASURE appears to split the distance between the \( X \), the CURRENT SAMPLE INDEX, and the circle, the LONG RUN AVERAGE.

**Percentile Indices**

Look at the ‘I’ Plots in Illustration 3. The two end points of the vertical line segments are called the 95th and 5th percentile indices, respectively. There is a 95% chance that the true (but unknown) quality is worse than the 95th percentile index. And, there is a 5% chance that the true (but unknown) quality is worse than the 5th percentile index. Therefore, in the figure labeled “A”, there is a 90% chance that the true (but unknown) quality falls within the interval. Note that this interval includes the Standard Quality value of 1. The “I” Plot gives the reader an interval estimate for CURRENT QUALITY.

**ILLUSTRATION 3**
Interpretation of Quality Trend Charts

A number of factors affects the interpretation of the Trend Chart results. In the next few pages, we’ll outline major elements you should look for in interpreting a chart.

Location of the “I” Plot

The location of the “I” Plot on the index scale is very important in the interpretation of a Trend Chart. We saw in figure “A” in Illustration 3 a case where the “I” Plot included the Standard Quality value of 1. In this case, we do not have strong evidence (there is less than a 95% chance) that the true quality is worse than the Standard Quality. On the other hand, in the figure labeled “B” in Illustration 3, the entire “I” Plot is greater than 1 and has fallen below the line representing Standard Quality. In such a case where the 95th percentile index falls below Standard Quality, and is greater than the value 1, we say that there is more than a 95% chance that the product or service tested is worse than Standard Quality. This is strong evidence of a quality problem.

Occasionally, the “I” Plot will exceed the boundaries of the chart. When this occurs, the numerical value of the 95th percentile index is printed on the Trend Chart for that period so that the reader has some indication of just how much worse than Standard Quality the CURRENT QUALITY really is that period. (See Illustration 4.)

ILLUSTRATION 4
Movement Over Time

Trend Charts are time series graphs, which show movement over several rating periods. Illustration 5 demonstrates how the three types of symbols on the “I” Plot move over time. The circle, which represents the LONG RUN AVERAGE, exhibits relatively little fluctuation from period to period, since the LONG RUN AVERAGE is a moving average of 6 periods of data. Upward movement indicates a long-run improvement in the process. The circles are connected, giving a strong indication of trend. The X, which represents the CURRENT SAMPLE INDEX, tends to fluctuate much more than the circle, primarily since it represents only one period of data (from the current period sample).

N Symbol—The N which appears in Period 9 in Illustration 5 indicates that no sample data were taken. In this case, the LONG RUN AVERAGE remains the same as in the previous period.

ILLUSTRATION 5

Dynamics of Sudden Change

Since results are calculated using 6 periods of data, an important area of concern is the degree of responsiveness of the “I” Plot to sudden changes in quality. A prior history of standard or substandard quality should not prevent the recognition of a sudden change in quality.

If a chronic quality problem is solved, then the plan should reflect this. Conversely, if there is a sudden degradation in quality, a good quality measurement plan should also detect it. However, the plan should be able to distinguish between a real change in the quality level and sampling variation. Illustration 6 shows the reaction of the Trend Charts to sudden changes in quality.
Sudden Degradation of Quality

Illustration 6 portrays both a sudden degradation and a sudden improvement in quality. In period 2, the CURRENT SAMPLE INDEX is approximately 1.5, indicating that about one and a half times as many defects were detected in the current period sample as were expected. Although the CURRENT SAMPLE INDEX seems to indicate that the current population quality is worse than the expected Standard Quality, QMP statistics show that this change in the sample result is explainable by sampling variation, and the product continues to be rated at Standard Quality. This shift is not significant enough to indicate a change in process quality, since the “T” Plot includes the Standard Quality value of 1.

ILLUSTRATION 6

From rating periods 2 to 3, we see a sudden downward shift in quality. In period 3, the CURRENT SAMPLE INDEX is approximately 3.2. Here QMP finds that the change is not due to sampling variation, because the entire “T” Plot has fallen below the Standard Quality line. Thus we can say that we have 95% confidence that the product is not meeting Standard Quality.

Sudden Improvement in Quality

There is a significant improvement in quality from periods 5 to 6 in Illustration 6. Quality had been worse than Standard Quality for three periods. Then, in period 6, there is a dramatic rise shown by the CURRENT SAMPLE INDEX (the X) moving to better than Standard Quality. There is a similar improvement in the BEST MEASURE OF CURRENT QUALITY (the dash). In period 6, there is no longer much evidence that the true quality is worse than Standard Quality. If the CURRENT SAMPLE INDEX stays at or better than Standard Quality in the next few periods, both the LONG RUN AVERAGE and the BEST MEASURE will gradually move up to Standard Quality or better.
The Length of Each "I" Plot

The length of each "I" Plot is a measure of the precision of the BEST MEASURE. Look at the sample "I" Plots in Illustration 7 and see how they reflect this concept. The "I" Plot on the left is very short, showing that there is little uncertainty about the quality in this rating. The range from the 5th to the 95th percentile indices is very small. The "I" Plot on the right, however, has a wide range of likely values for the true quality from the 5th to the 95th percentile indices. Thus, there is more uncertainty.

ILLUSTRATION 7

The precision depends on several factors, one of which is the size of the sample taken. In any sample, complete accuracy can never be totally assured, unless the entire population is inspected. Since we can't test the whole population, there is uncertainty in our result; however, the more items that are sampled, the better the evaluation.

EXPECTANCY is a better measure than sample size of the amount of information in the sample. EXPECTANCY is the number of defects expected at Standard Quality. It is defined as the sample size, n, times the Standard Quality per unit, s (i.e., EXPECTANCY = n x s). It allows a more equitable comparison of Trend Charts for different products, where the sample sizes may be the same but the Standard Quality per unit is different, as shown by the following example. A sample of 100 switching systems has a relatively large expectancy as compared to a sample of 100 circuit packs, which has a much smaller expectancy. We obtain a lot more information from inspecting 100 switching systems than from inspecting 100 circuit packs even though the sample sizes are the same.

The length of the "I" Plot represents the precision of the BEST MEASURE. EXPECTANCY is one of the factors which influence the precision of the BEST MEASURE. The smaller the EXPECTANCY, the longer the "I" Plot, and the greater the uncertainty about quality. The larger the EXPECTANCY, the shorter the "I" Plot, and thus the greater the precision in our estimate of quality.
The length of the ‘T’ Plot actually depends on an intricate relationship between the size of the expectancy (or sample size) and two other factors: process stability and the BEST MEASURE OF CURRENT QUALITY. Although the relationship between these factors is quite complex, a change in the length of the ‘T’ Plot is generally attributable to one, or a combination, of these three conditions:

<table>
<thead>
<tr>
<th>CHANGE IN ‘T’ PLOT LENGTH</th>
<th>PROCESS STABILITY</th>
<th>BEST MEASURE OF CURRENT QUALITY</th>
<th>EXPECTANCY (SAMPLE SIZE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shorter</td>
<td>Steady</td>
<td>Improved</td>
<td>Large</td>
</tr>
<tr>
<td>Longer</td>
<td>Erratic</td>
<td>Deteriorated</td>
<td>Small</td>
</tr>
</tbody>
</table>

A more concise way to view this relationship is as follows. The length of the ‘T’ Plot is proportional to

\[
\sqrt{\frac{PV}{SV + PV}} \cdot \frac{\text{BEST MEASURE}}{\text{EXPECTANCY}}
\]

In Illustration 5 (see page 10), notice the change in the length of the ‘T’ Plot from Period 3 to Period 4. The ‘T’ Plot gets shorter, probably due to an improvement in the BEST MEASURE and possibly a larger expectancy.
Quality Trend Chart Examples

The following examples offer a variety of different types of process quality behavior. It is hoped that looking at these example Trend Charts and the attached interpretations will provide assistance in reviewing the Trend Charts periodically distributed by Bellcore-QA.

EXAMPLE 1
This is an example of a well controlled process where quality is at Standard Quality. There is very little process variation.

EXAMPLE 2
This is an example of a well controlled process. There is very little variation, process or sampling variation, in the process. It is a very stable process that is running much better than Standard Quality. From May through November there were no defects observed, as shown by a zero CURRENT SAMPLE INDEX.
EXAMPLE 3
This is an example of an unstable process, a process with much sampling variability as well as process variability. Each period the interval for CURRENT QUALITY is shifting significantly. This interval bounces from better than Standard Quality to Standard Quality to worse than Standard Quality, etc. It is process variability that causes this fluctuation. The trend during the last five periods is around 1.5 times Standard Quality even though period 10 shows a BEST MEASURE better than Standard Quality. This process has a quality problem.

EXAMPLE 4
This process is literally off the chart. The numbers at the bottom indicate that the entire interval estimate for CURRENT QUALITY is worse than 5 or 6 indicating that CURRENT QUALITY is very likely worse than 5 or 6 times Standard Quality. This represents a severe quality problem.
EXAMPLE 5

During the periods 2-4, the process in this example was running at better than Standard Quality and during the periods 1 and 5-6 the process was running at Standard Quality. From period 5 on there is more variation in the process and more sampling variation as the CURRENT SAMPLE INDEX bounces around. Even though there is insufficient statistical evidence to say with 95% confidence that quality is worse than Standard Quality, there is a degradation, or trend, in quality, and by period 9 we can say with 95% confidence that quality is worse than Standard Quality.

EXAMPLE 6

This process quickly drifted to between 1.5 and 2 times Standard Quality and shows considerable sampling variation. Note that the interval estimate for CURRENT QUALITY is entirely worse than Standard Quality in periods 4 and 10 indicating a quality problem. During Periods 5 through 9, the CURRENT QUALITY is probably worse than Standard Quality at least part of the time. This is true even though the evidence does not exceed 95% for any single period from 5 through 9, because all the interval estimates include the Standard Quality value of 1.
EXAMPLE 7

This is a well controlled process. Very few defects have ever been observed. No data were collected for September or October; resources are used for other products where needed. However, an occasional check is needed to verify that the process has not changed significantly. Data were collected in November.

EXAMPLE 8

This example shows considerable sampling variation. Except for period 2, there is insufficient evidence to indicate quality for any particular period is worse than Standard Quality until November. But the LONG RUN AVERAGE shows quality between 1 and 2 times Standard Quality most of the year. The recent trend in the LONG RUN AVERAGE from August through November, from Standard Quality to almost 2 times Standard Quality and the fact that quality in November is worse than Standard Quality should cause the reader to be seriously concerned about the quality of product to be produced in December.

Why did the LONG RUN AVERAGE get larger in Period 10 when there were fewer defects than period 9 (as indicated by the CURRENT SAMPLE INDEX)? Recall that the LONG RUN AVERAGE is based on a window of 6 periods. Note that when period 10 was added (with some defects), period 4 (with no defects) dropped out of the window. The ‘T’ Plot for period 10 is also shorter than in most previous periods, meaning that a larger sample was taken in period 10 (than period 4 or 5 or 7 which have no defects). Thus more weight is given to period 10 in the current LONG RUN AVERAGE than was given to period 4 in the previous LONG RUN AVERAGE.
Reference List


If further information regarding the development and maintenance of the methodology used in the Trend Charts is required, please contact:

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